EE 505
Lecture 6

Spectral Analysis in Spectre

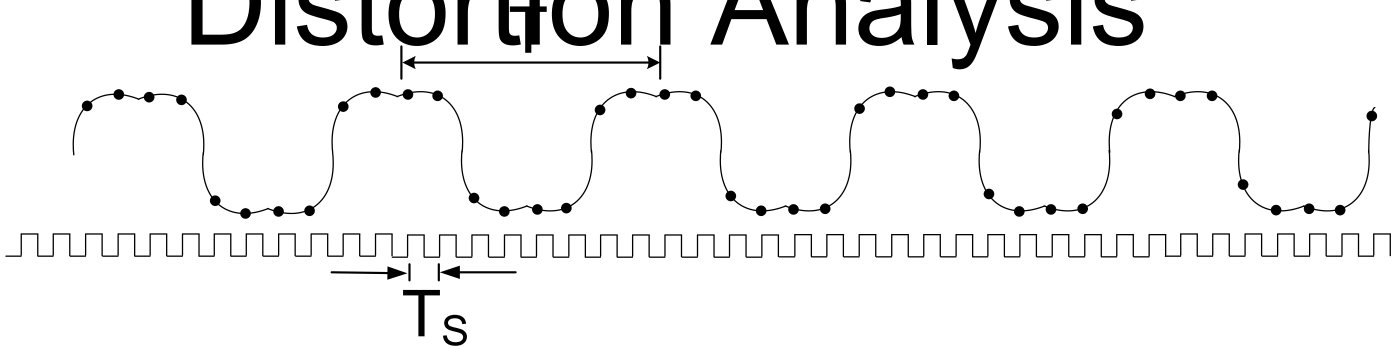
- Standard transient analysis
- Strobe period transient analysis

Addressing Spectral Analysis Challenges

- Problem Awareness
- Windowing
- Post-processing

Review from Last Lecture

Distortion Analysis



THEOREM: Consider a periodic signal with period $T=1/f$ and sampling period $T_S=1/f_S$. If N_P is an integer, $x(t)$ is band limited to f_{MAX}, and $f_s>2f_{max}$, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad 0 \le m \le h-1$$

and $X(k)=0$ for all k not defined above

where $\left\langle X(k)\right\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\left\langle x(kT_S)\right\rangle_{k=0}^{N-1}$

N=number of samples, N_P is the number of periods, and $h = \mathrm{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

N_P an integer means $N_P=N\frac{T_S}{T}$ is an integer

Spectral components of interest are $|A_m|$, m=0….h-1

Key Theorem central to Spectral Analysis that is widely used !!! and often “abused”

Review from Last Lecture

Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
- Windowing

Tool Validation (MATLAB)

Likely does not cause significant errors for existing data converter spectral characterization applications

Likely can't attribute unexpected results in a design to MATLAB limitations for spectral characterization

Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
- Windowing

Considerations for Spectral Characterization

FFT Length

- FFT Length does not significantly affect the computational noise floor
- Although not shown here yet, FFT length does reduce the quantization noise floor coefficients

If we assume E_{QUANT} is fixed

$$E_{QUANT} \cong \sqrt{\sum_{k=2}^{2^{n_{DFT}}} A_k^2}$$

If the A_k's are constant and equal

$$E_{QUANT} \cong A_k 2^{n_{DFT}/2}$$

Solving for A_k, obtain

$$A_k \cong \frac{E_{QUANT}}{2^{n_{DFT}/2}}$$

If input is full-scale sinusoid with only amplitude quantization with n-bit res,

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

Review from Last Lecture

Considerations for Spectral Characterization

FFT Length

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

Substituting for E_{QUANT}, obtain

$$A_k \cong \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1} 2^{n_{DFT}/2}}$$

This value for A_k thus decreases with the length of the DFT window

Example: if n=16, n_{DFT}=12 (4096 pt transform), and X_{REF}=1V,
then A_k=6.9E-8V (-143dB),

(Note A_k>> computational noise for all practical n, n_{DFT})

Review from Last Lecture

Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing

Example

WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

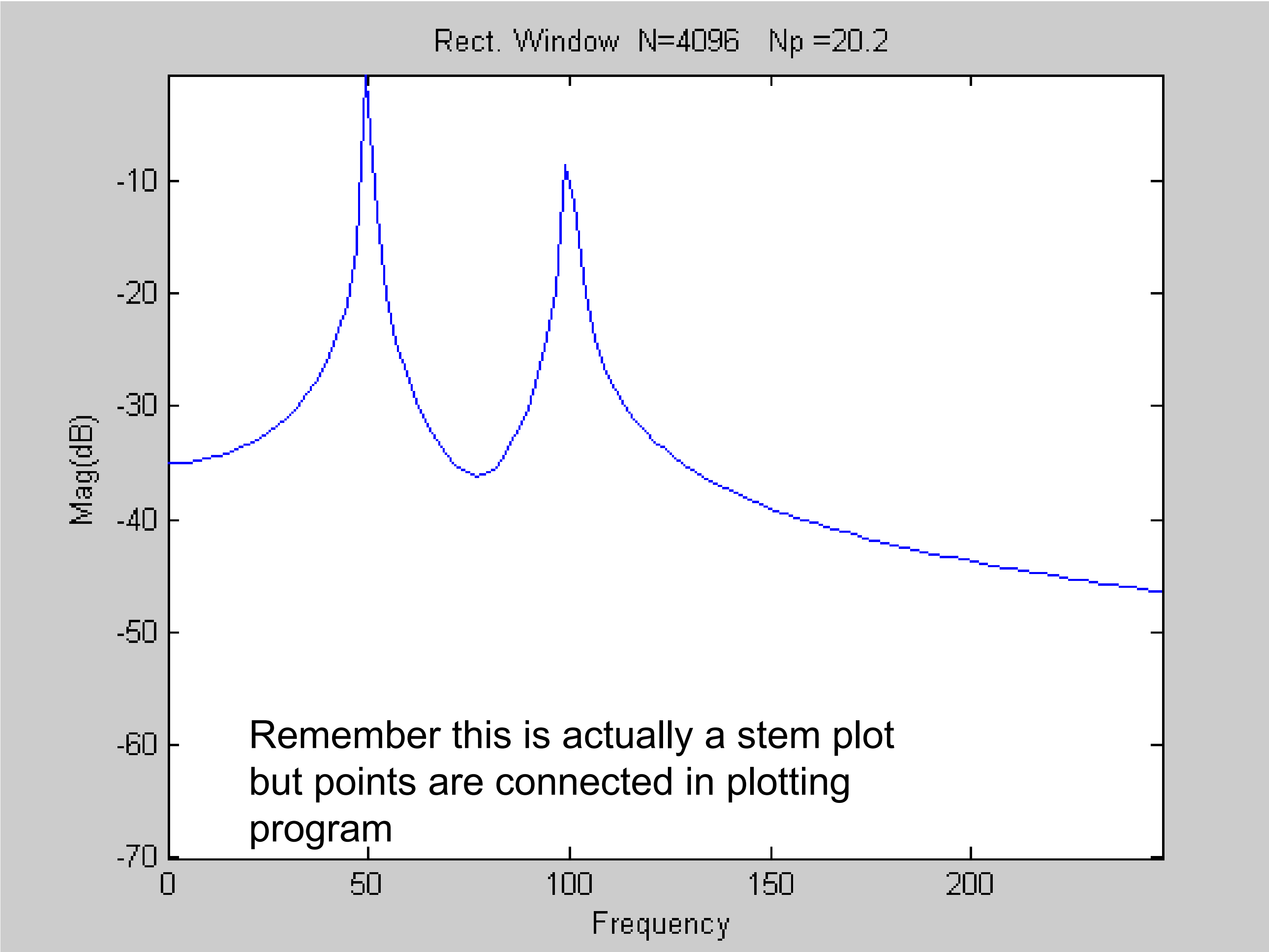
$$\omega = 2\pi f_{SIG}$$

Consider N_P=20.2 N=4096

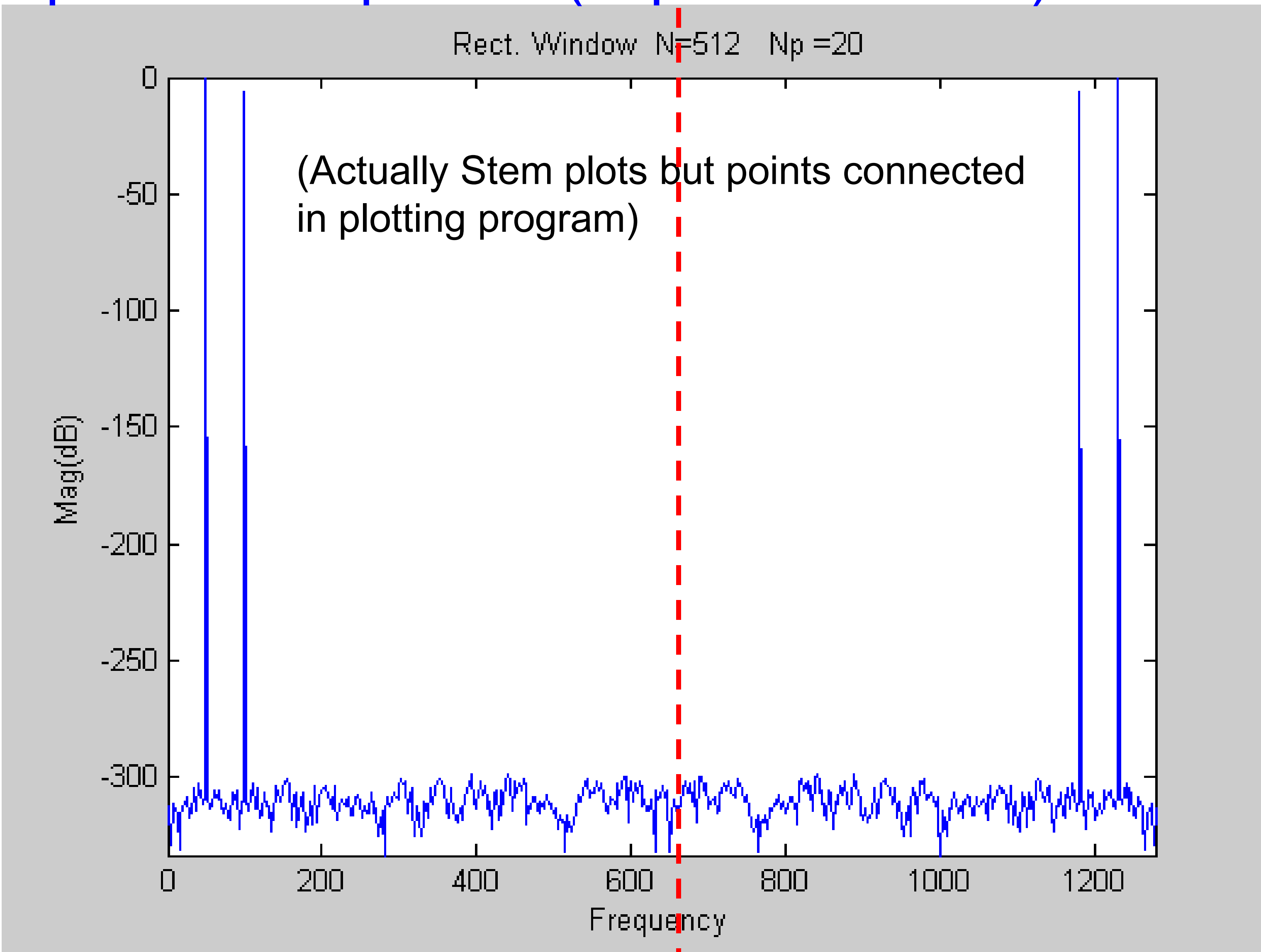
Recall $20\log_{10}(0.5)=-6.0205999$

Review from Last Lecture

Spectral Response



Spectral Response (expressed in dB)

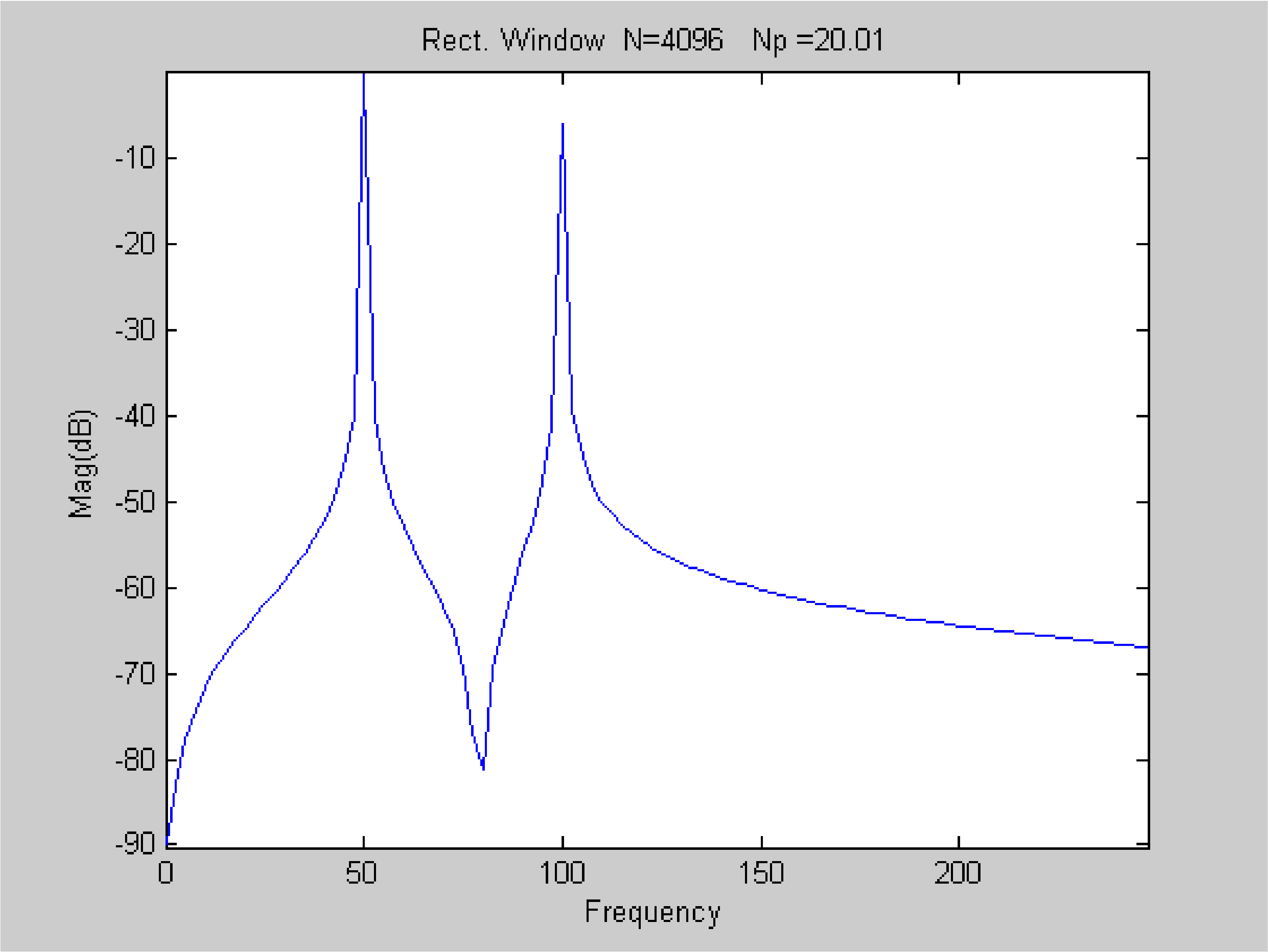


Note Magnitude is Symmetric wrt $0.5*f_{SAMPLE}$

$$f_{AXIS} = f_{SIGNAL} \frac{n-1}{N_P}$$

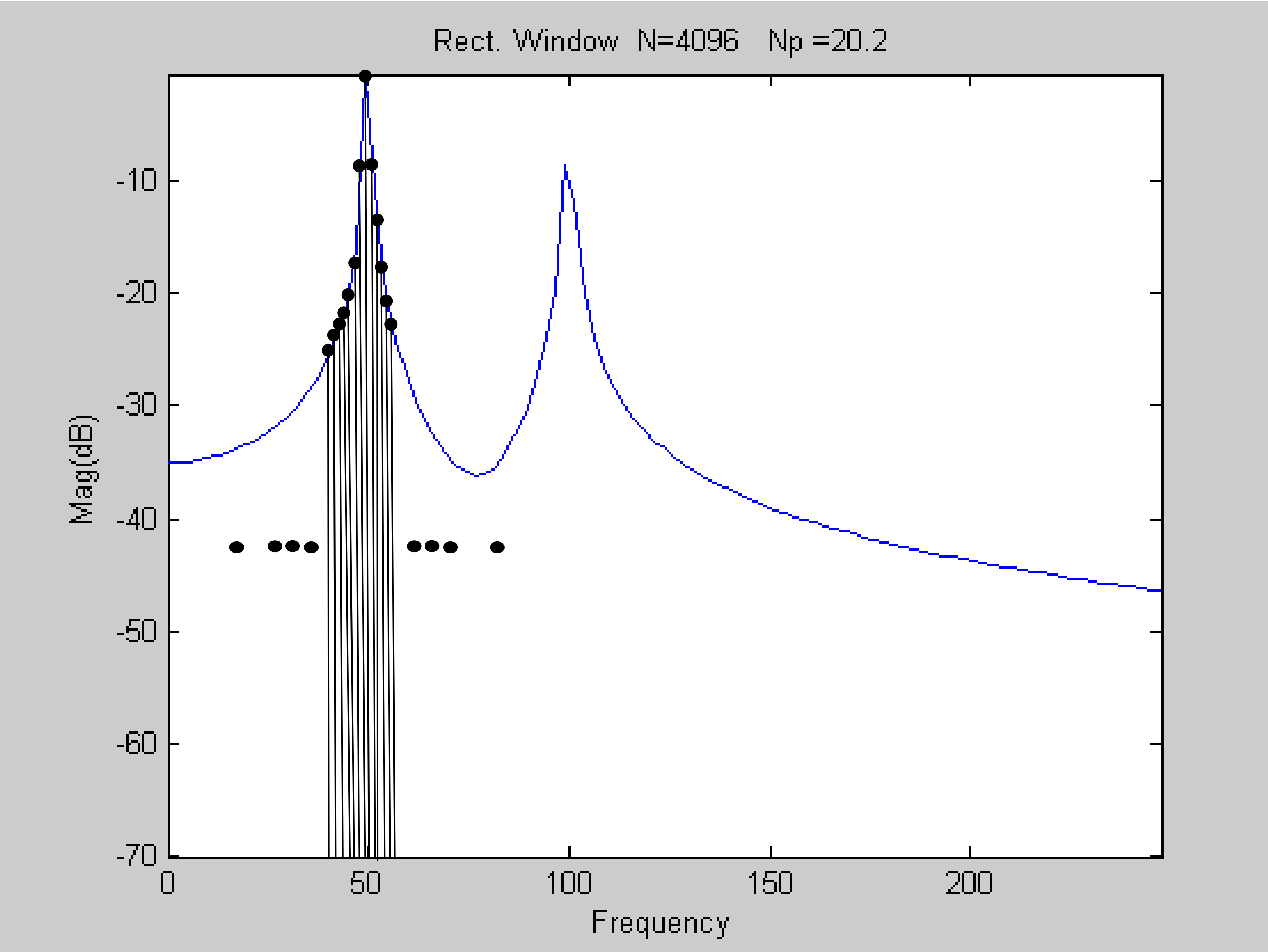
Review from last lecture . . . .

Spectral Response with Non-coherent Sampling



(zoomed in around fundamental)

Spectral Response



Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-35.0366 -35.0125 -34.9400 -34.8182 -34.6458 -34.4208 -34.1403

Columns 8 through 14

-33.8005 -33.3963 -32.9206 -32.3642 -31.7144 -30.9535 -30.0563

Columns 15 through 21

-28.9855 -27.6830 -26.0523 -23.9155 -20.8888 -15.8561 **-0.5309**

Columns 22 through 28

-12.8167 -20.1124 -24.2085 -27.1229 -29.4104 -31.2957 -32.8782

Columns 29 through 35

-34.1902 -35.2163 -35.9043 -36.1838 -35.9965 -35.3255 -34.1946

Note there is a dramatic increase in the noise floor and a significant change in and spreading of the fundamental!!

k^{th} harmonic will appear at position 1+k•Np

Columns 36 through 42

-32.6350 -30.6397 -28.1125 -24.7689 -19.7626 -8.5639 -11.7825

Columns 43 through 49

-20.0158 -23.9648 -26.5412 -28.4370 -29.9279 -31.1519 -32.1874

Columns 50 through 56

-33.0833 -33.8720 -34.5759 -35.2113 -35.7902 -36.3218 -36.8133

Columns 57 through 63

-37.2703 -37.6974 -38.0984 -38.4762 -38.8336 -39.1725 -39.4949

Columns 64 through 70

-39.8024 -40.0963 -40.3778 -40.6479 -40.9076 -41.1576 -41.3987

k^{th} harmonic will appear at position 1+k•Np

Columns 36 through 42

-32.6350 -30.6397 -28.1125 -24.7689 -19.7626 -8.5639 -11.7825

Columns 43 through 49

-20.0158 -23.9648 -26.5412 -28.4370 -29.9279 -31.1519 -32.1874

Columns 50 through 56

-33.0833 -33.8720 -34.5759 -35.2113 -35.7902 -36.3218 -36.8133

Columns 57 through 63

-37.2703 -37.6974 -38.0984 -38.4762 -38.8336 -39.1725 -39.4949

Columns 64 through 70

-39.8024 -40.0963 -40.3778 -40.6479 -40.9076 -41.1576 -41.3987

Observations

- Modest change in sampling window of 0.2 out of 20 periods (1%) results in a big error in both fundamental and harmonic
- More importantly, dramatic raise in the "noise floor" !!! (from over -300dB to only -12dB)

Example

WLOG assume f_{SIG}=50Hz

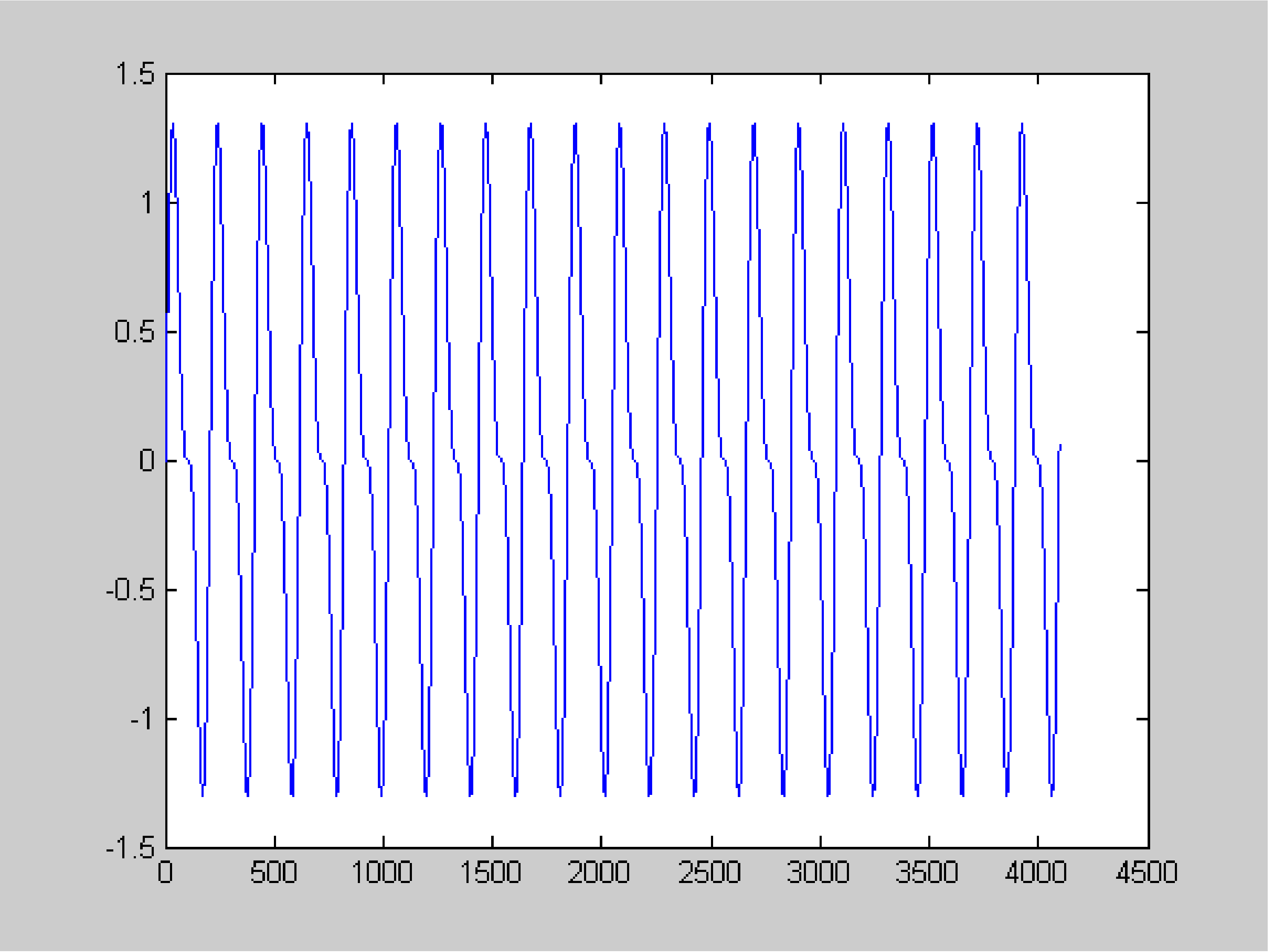
$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

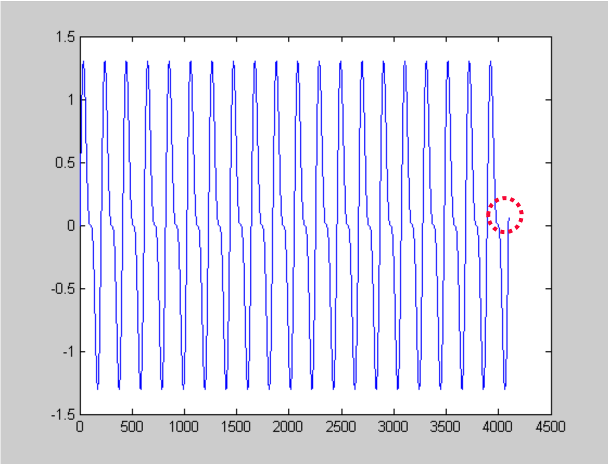
Consider N_P=20.01 N=4096

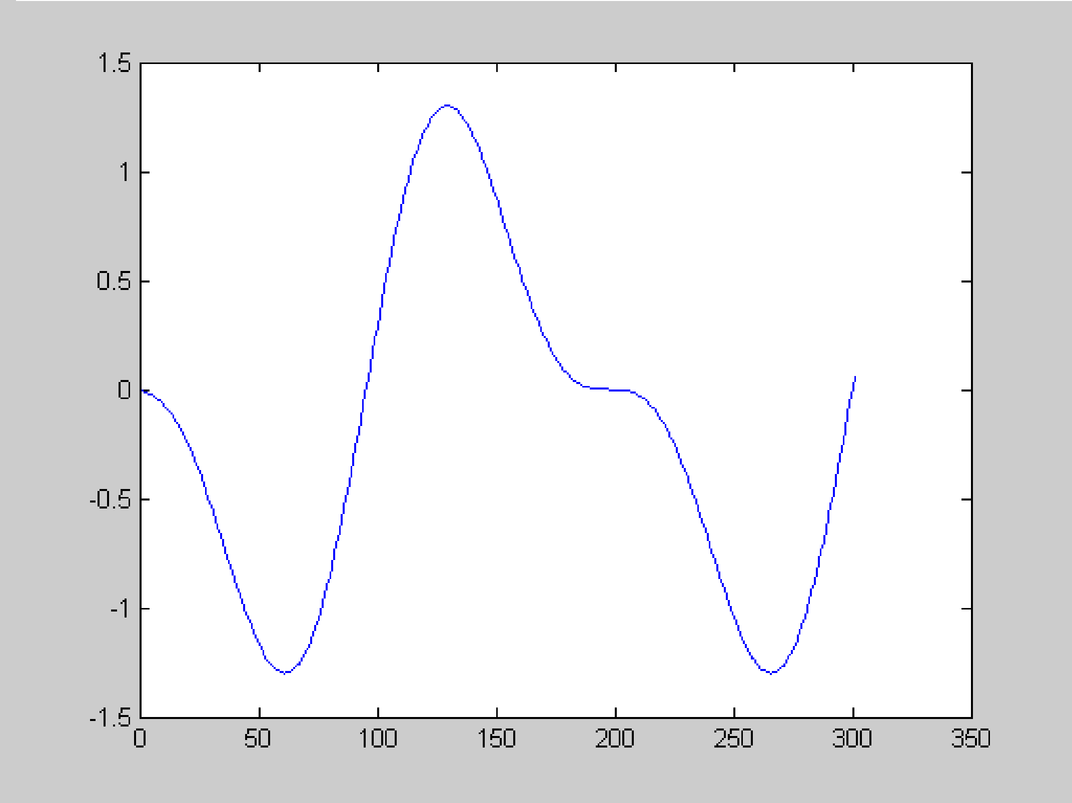
Deviation from hypothesis is .05% of the sampling window

Input Waveform

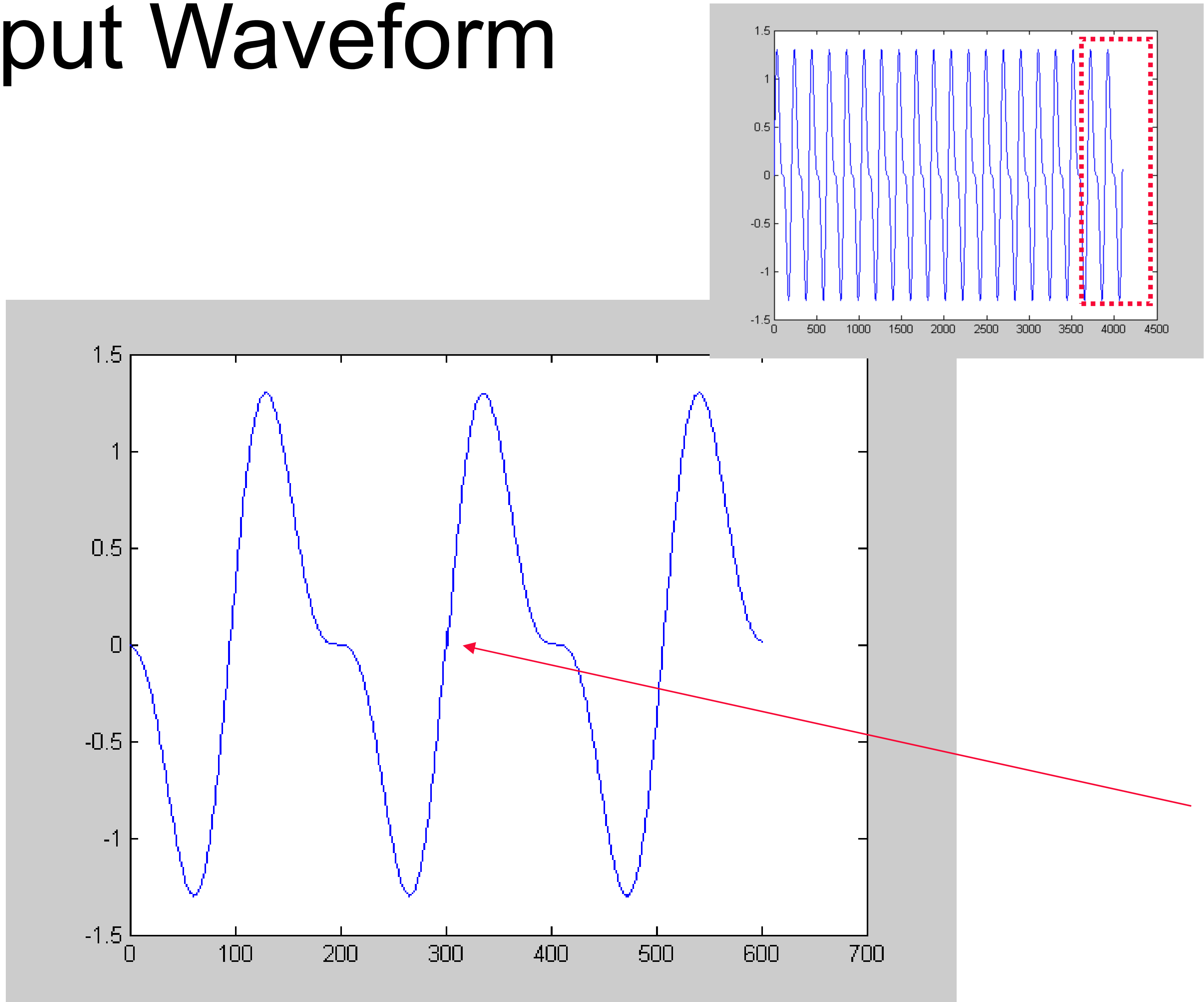


Input Waveform

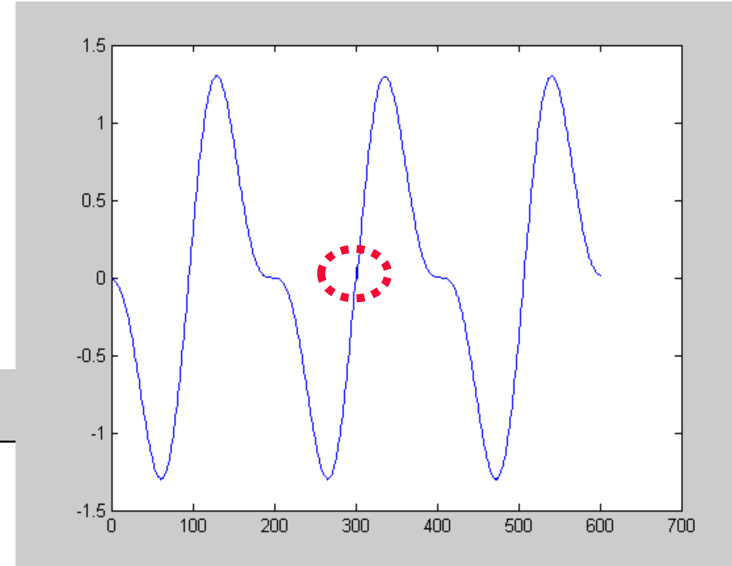


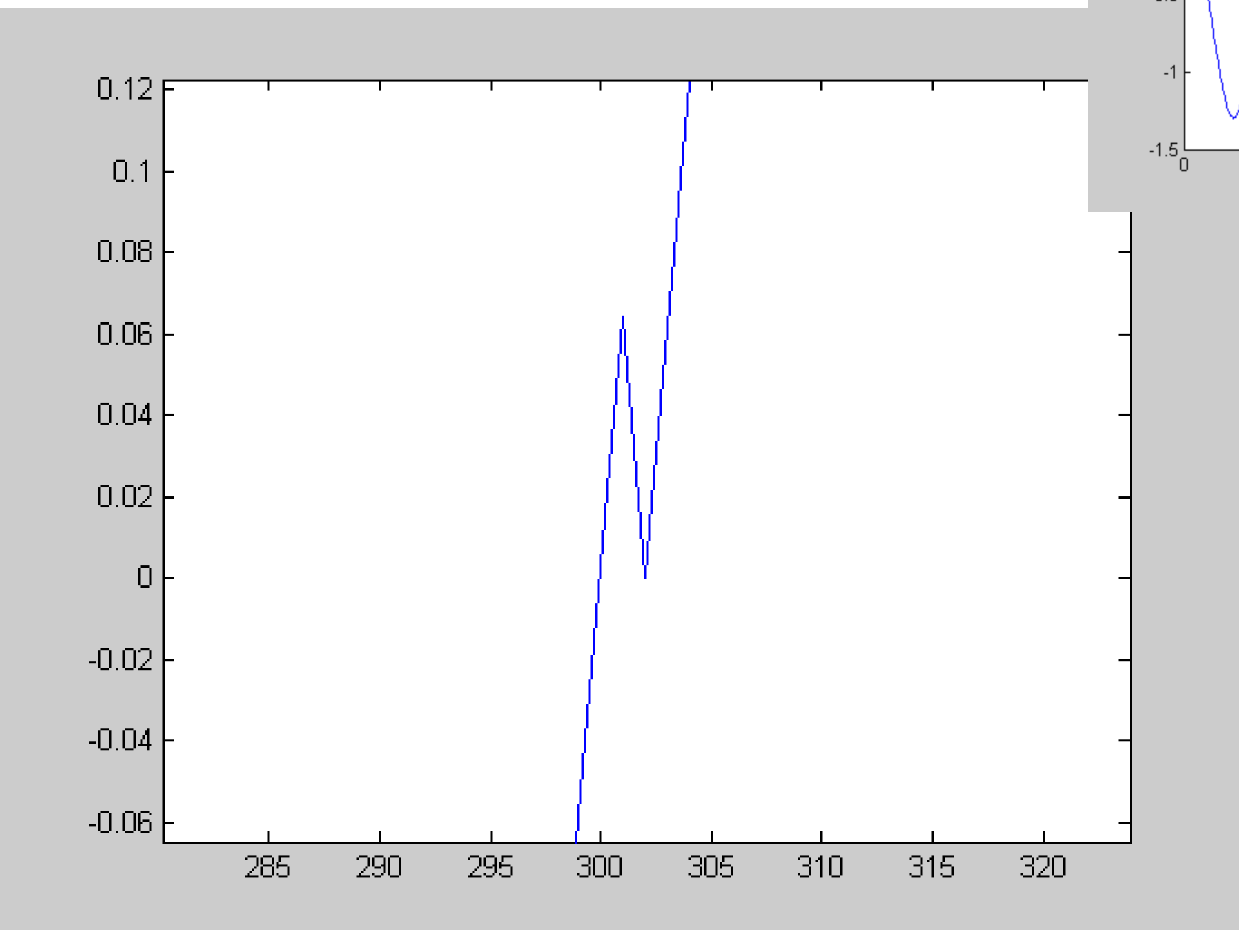


Input Waveform

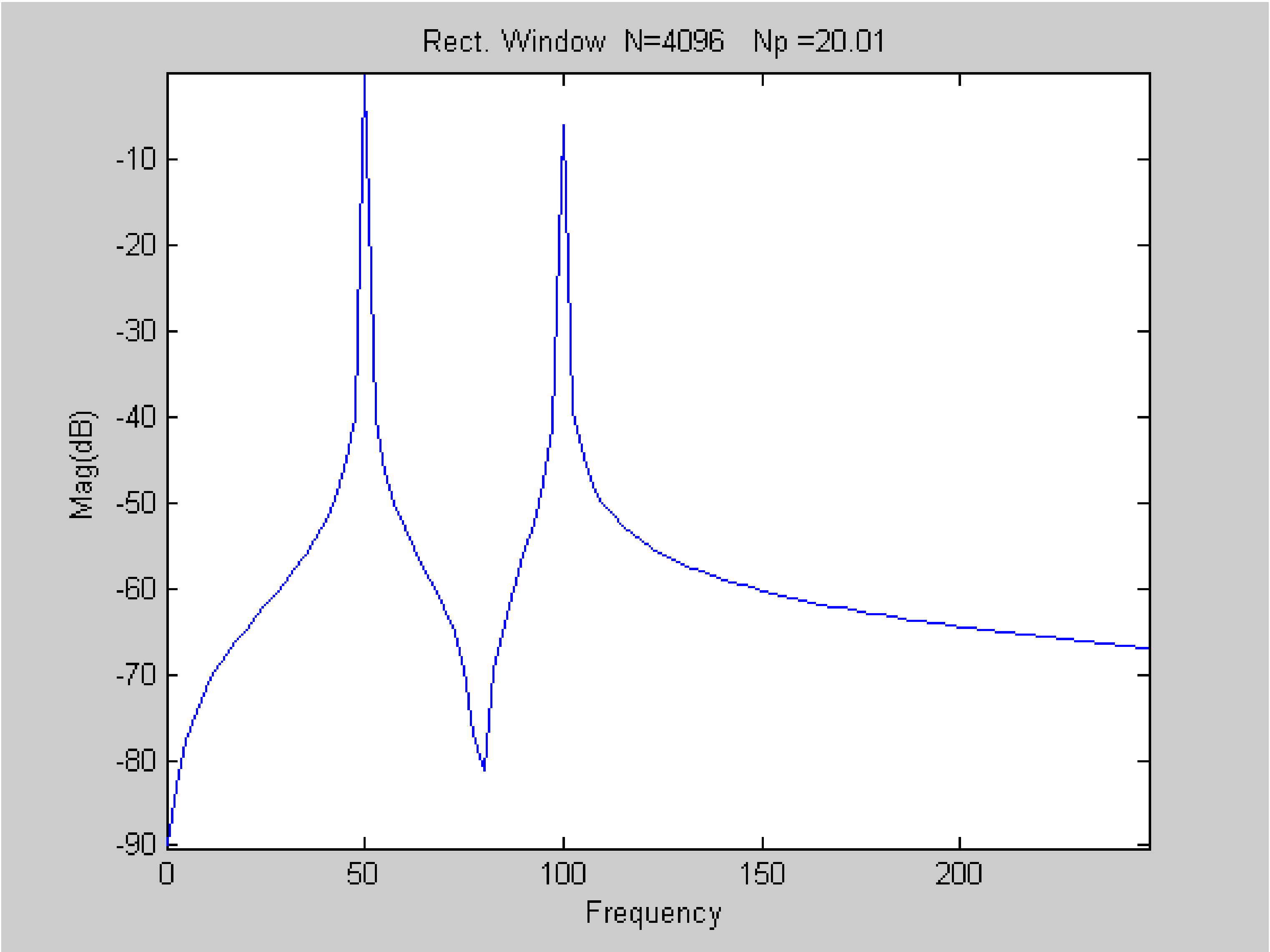


Input Waveform





Spectral Response with Non-Coherent Sampling



(zoomed in around fundamental)

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-89.8679 -83.0583 -77.7239 -74.2607 -71.6830 -69.5948 -67.8044

Columns 8 through 14

-66.2037 -64.7240 -63.3167 -61.9435 -60.5707 -59.1642 -57.6859

Columns 15 through 21

-56.0866 -54.2966 -52.2035 -49.6015 -46.0326 -40.0441 -0.0007

Columns 22 through 28

-40.0162 -46.2516 -50.0399 -52.8973 -55.3185 -57.5543 -59.7864

Columns 29 through 35

-62.2078 -65.1175 -69.1845 -76.9560 -81.1539 -69.6230 -64.0636

k^{th} harmonic will appear at position $1+k \cdot Np$

Columns 36 through 42

-59.9172	-56.1859	-52.3380	-47.7624	-40.9389	-6.0401	-39.2033

Observations

- Modest change in sampling window of 0.01 out of 20 periods (.05%) still results in a modest error in both fundamental and harmonic
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -40dB)
- Errors at about the 6-bit level !

Example

WLOG assume f_{SIG}=50Hz

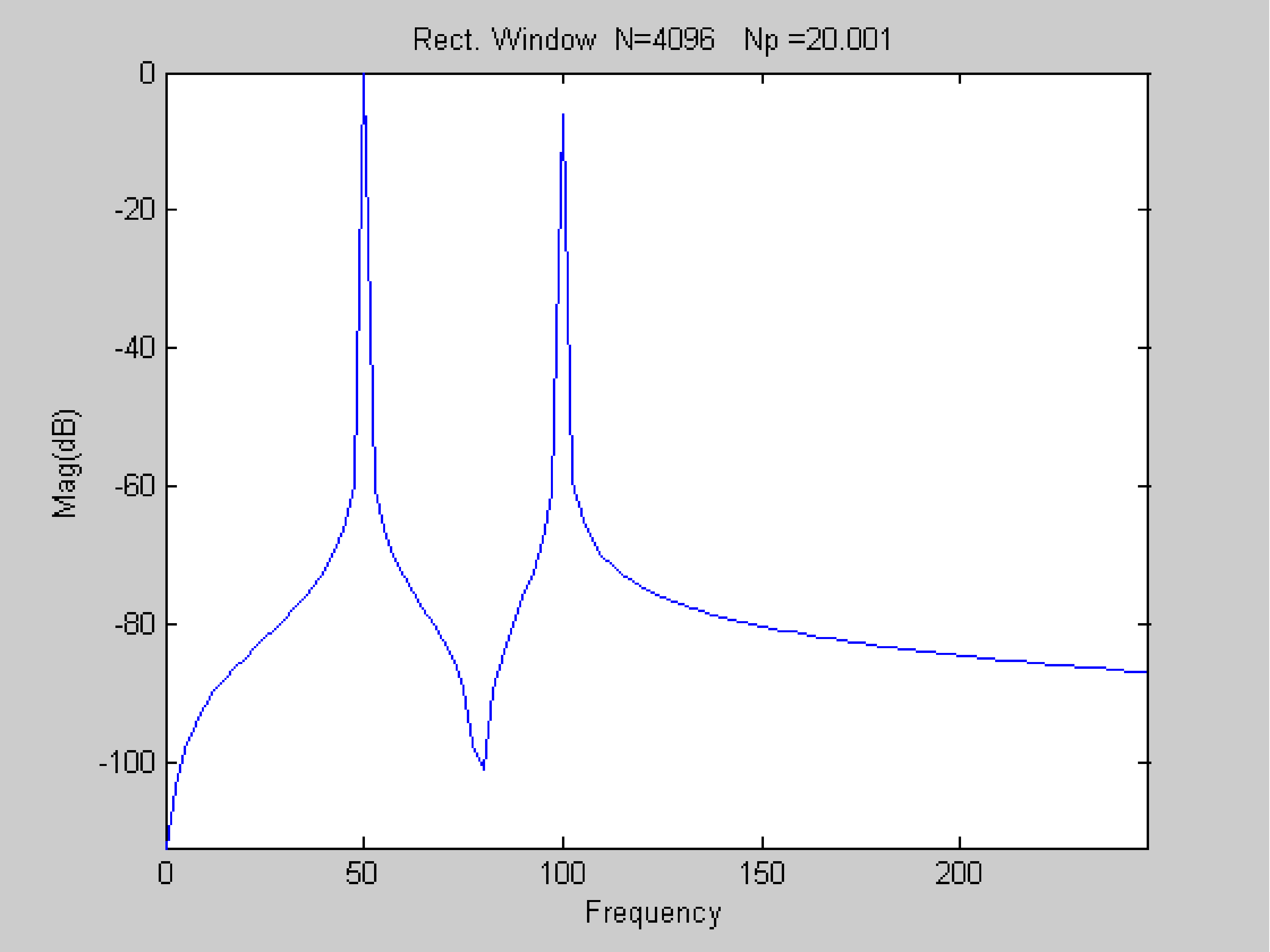
$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider N_P=20.001 N=4096

Deviation from hypothesis is .005% of the sampling window

Spectral Response with Non-coherent Sampling



(zoomed in around fundamental)

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-112.2531 -103.4507 -97.8283 -94.3021 -91.7015 -89.6024 -87.8059

Columns 8 through 14

-86.2014 -84.7190 -83.3097 -81.9349 -80.5605 -79.1526 -77.6726

Columns 15 through 21

-76.0714 -74.2787 -72.1818 -69.5735 -65.9919 -59.9650 0.0001

Columns 22 through 28

-60.0947 -66.2917 -70.0681 -72.9207 -75.3402 -77.5767 -79.8121

Columns 29 through 35

-82.2405 -85.1651 -89.2710 -97.2462 -101.0487 -89.5195 -83.9851

k^{th} harmonic will appear at position 1+k•Np

Columns 36 through 42

-79.8472 -76.1160 -72.2601 -67.6621 -60.7642 -6.0220 -59.3448

Columns 43 through 49

-64.8177 -67.8520 -69.9156 -71.4625 -72.6918 -73.7078 -74.5718

Columns 50 through 56

-75.3225 -75.9857 -76.5796 -77.1173 -77.6087 -78.0613 -78.4809

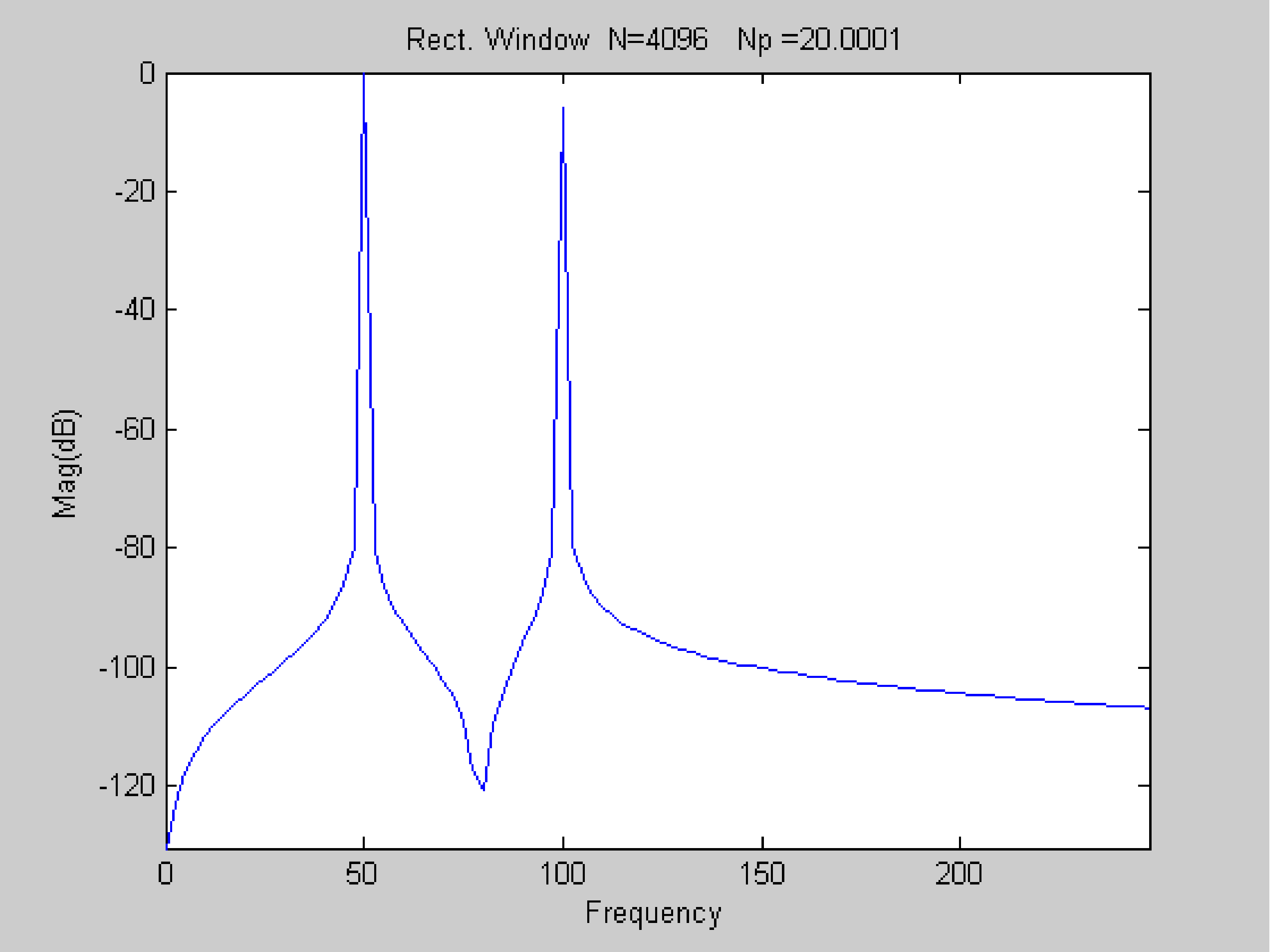
Columns 57 through 63

-78.8721 -79.2387 -79.5837 -79.9096 -80.2186 -80.5125 -80.7927

Observations

- Modest change in sampling window of 0.01 out of 20 periods (.005%) results in a small error in both fundamental and harmonic
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -60dB)
- Errors at about the 10-bit level !

Spectral Response with Non-coherent sampling



(zoomed in around fundamental)

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-130.4427 -123.1634 -117.7467 -114.2649 -111.6804 -109.5888 -107.7965

Columns 8 through 14

-106.1944 -104.7137 -103.3055 -101.9314 -100.5575 -99.1499 -97.6702

Columns 15 through 21

-96.0691 -94.2764 -92.1793 -89.5706 -85.9878 -79.9571 0.0000

Columns 22 through 28

-80.1027 -86.2959 -90.0712 -92.9232 -95.3425 -97.5788 -99.8141

Columns 29 through 35

-102.2424 -105.1665 -109.2693 -117.2013 -120.8396 -109.4934 -103.9724

k^{th} harmonic will appear at position 1+k•Np

Columns 36 through 42

-99.8382 -96.1082 -92.2521 -87.6522 -80.7470 -6.0207 -79.3595

Columns 43 through 49

-84.8247 -87.8566 -89.9190 -91.4652 -92.6940 -93.7098 -94.5736

Columns 50 through 56

-95.3241 -95.9872 -96.5810 -97.1187 -97.6100 -98.0625 -98.4821

Columns 57 through 63

-98.8732 -99.2398 -99.5847 -99.9107 -100.2197 -100.5135 -100.7937

Columns 64 through 70

Observations

- Modest change in sampling window of 0.001 out of 20 periods (.0005%) results in a small error in both fundamental and harmonic
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -80dB)
- Errors at about the 13-bit level !

Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing

Stay Safe and Stay Healthy !

FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. $$N > \frac{2f_{max}}{f_{SIGNAL}} N_P$$

Example

If f_{SIG}=50Hz

and N_P=20 N=512

$$f_{SAMP} = f_{SIG} \frac{N}{N_P}$$

f_{SAMP}=1280 Hz

$$N > \frac{2f_{max}}{f_{SIGNAL}} N_P \quad \Rightarrow \quad f_{max} < 640\text{Hz}$$

Example

Consider N_P=20 N=512

If f_{SIG}=50Hz

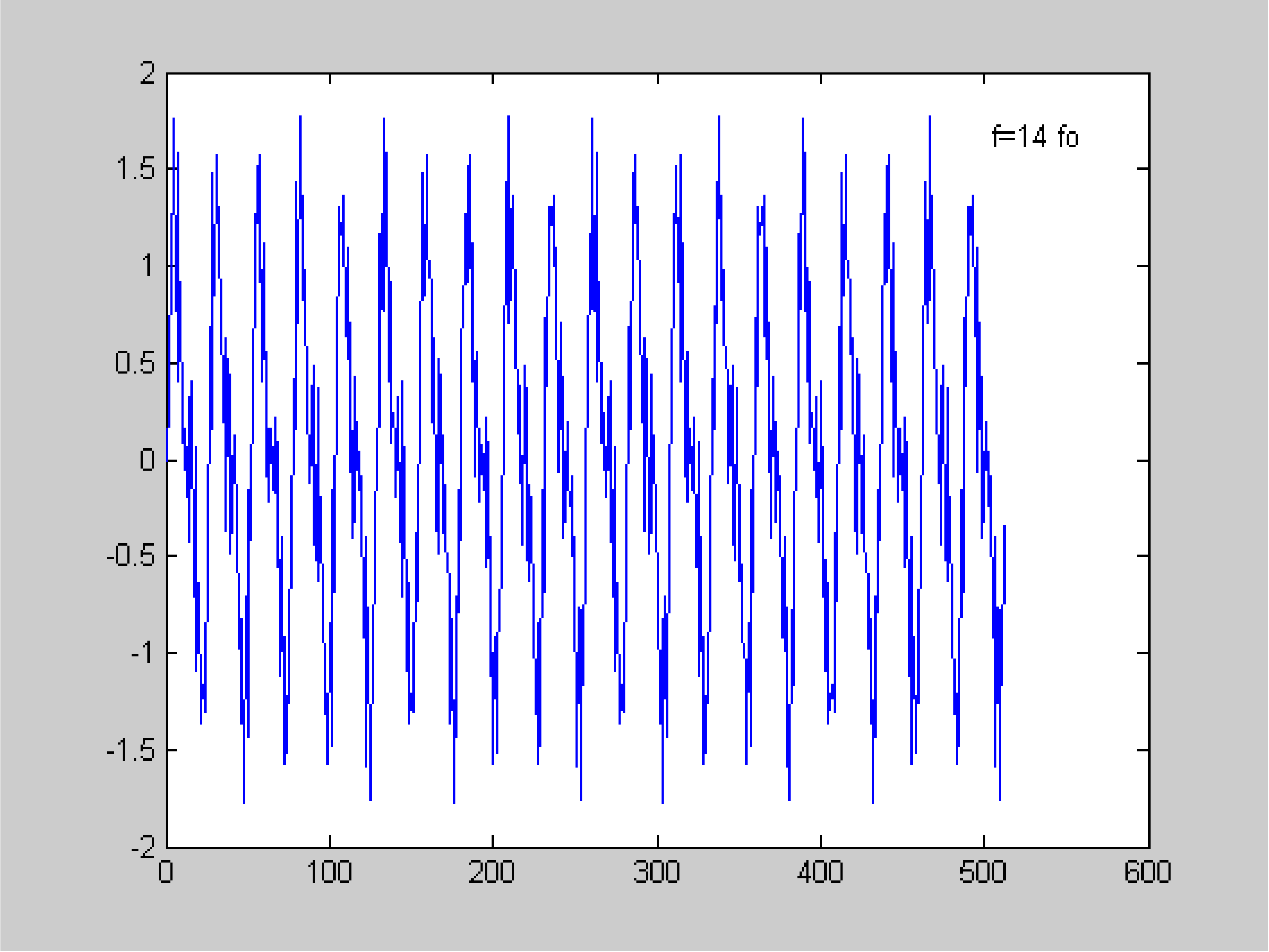
$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t) + 0.5\sin(14\omega t)$$

$$\omega = 2\pi f_{SIG}$$

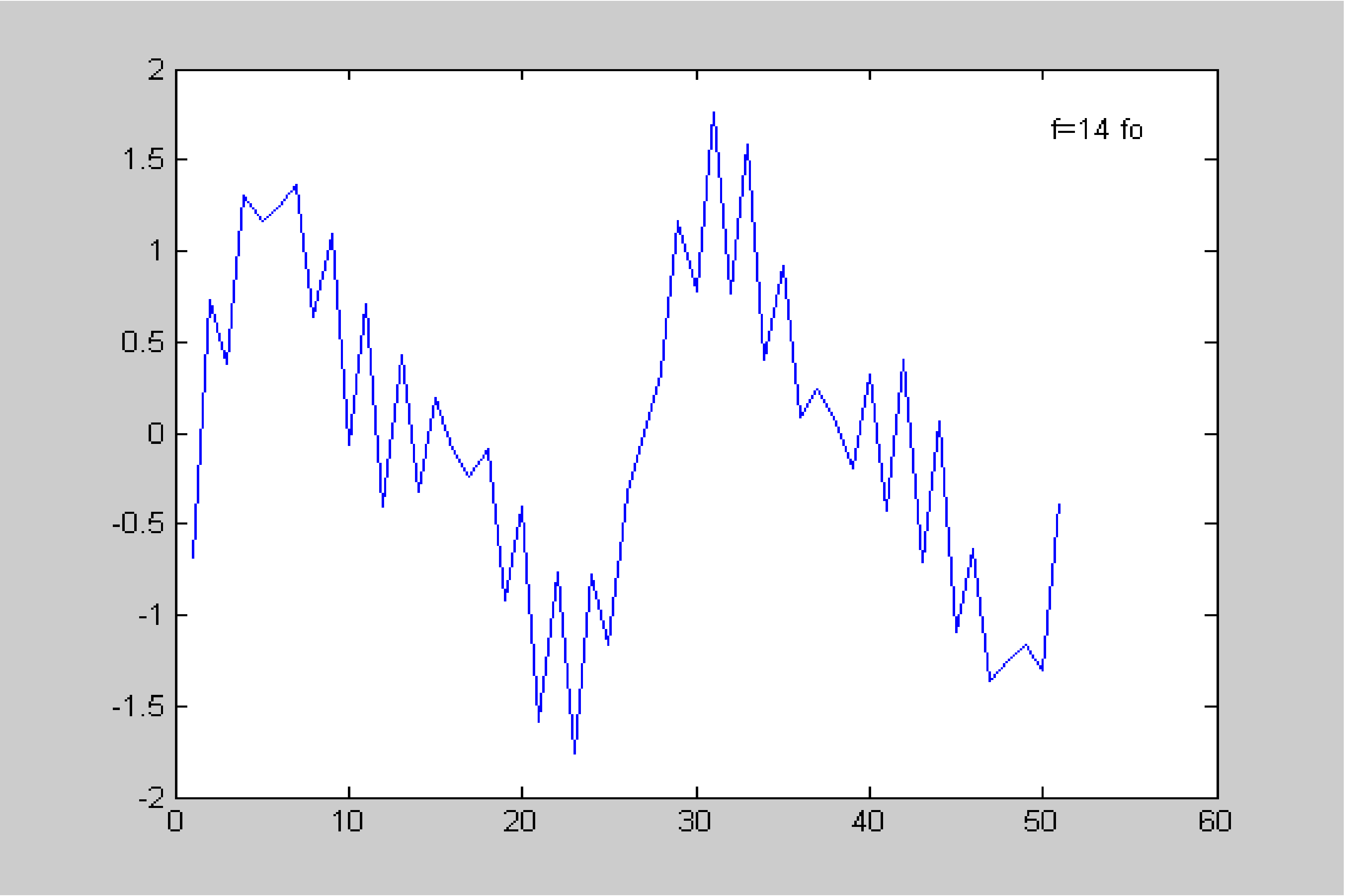
(i.e. a component at 700 Hz which violates the band limit requirement)

Recall $20\log_{10}(0.5)=-6.0205999$

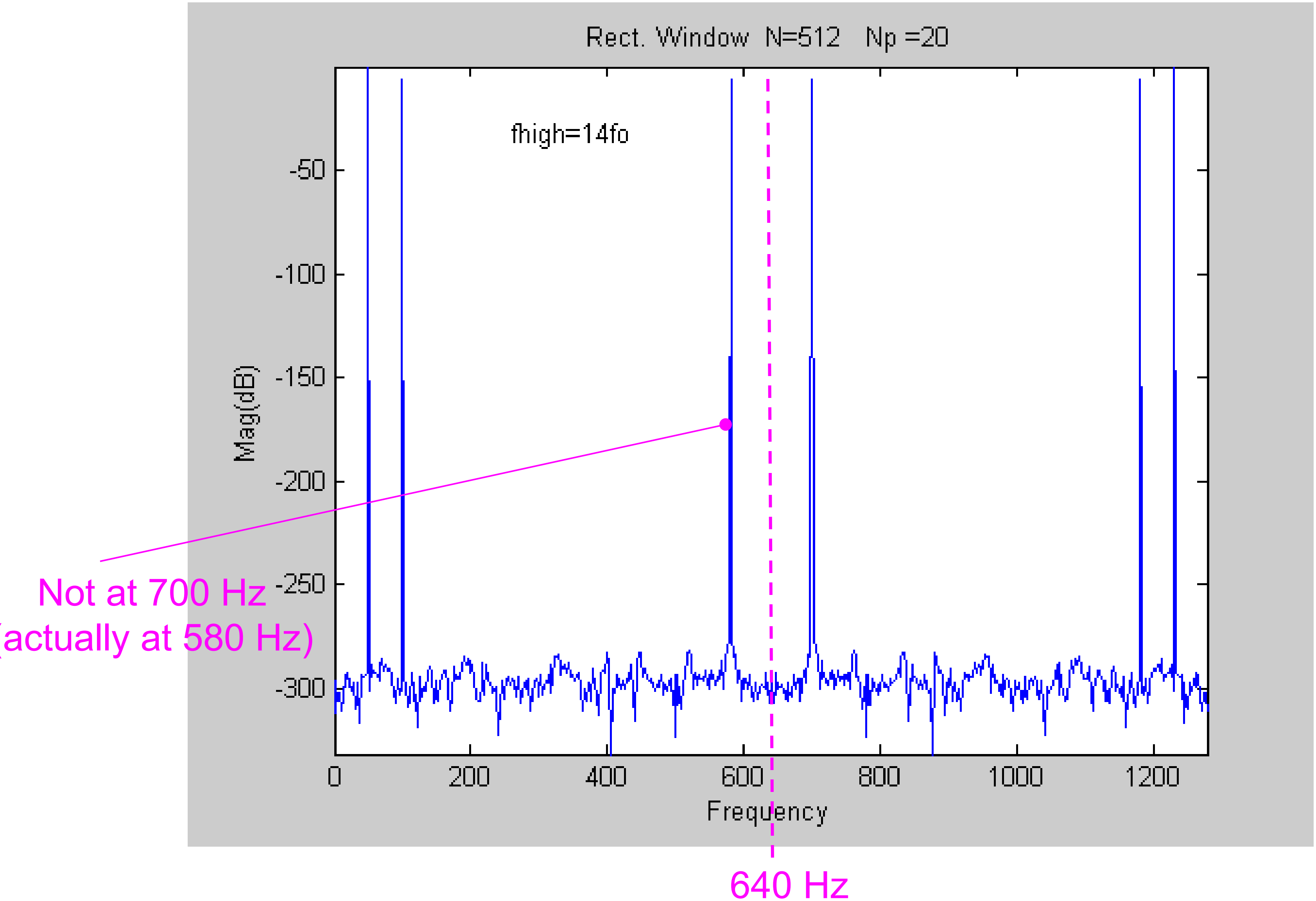
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

$f_{high}=14fo$

Columns 1 through 7

-296.9507 -311.9710 -302.4715 -302.1545 -310.8392 -304.5465 -293.9310

Columns 8 through 14

-299.0778 -292.3045 -297.0529 -301.4639 -297.3332 -309.6947 -308.2308

Columns 15 through 21

-297.3710 -316.5113 -293.5661 -294.4045 -293.6881 -292.6872 -0.0000

Columns 22 through 28

-301.6889 -288.4812 -292.5621 -292.5853 -294.1383 -296.4034 -289.5216

Columns 29 through 35

-285.9204 -292.1676 -289.0633 -292.1318 -290.6342 -293.2538 -296.8434

Effects of High-Frequency Spectral Components

f_{high}=14fo

Columns 36 through 42

-301.7087 -307.2119 -295.1726 -303.4403 -301.6427 -6.0206 -295.3018

Columns 43 through 49

-298.9215 -309.4829 -306.7363 -293.0808 -300.0882 -306.5530 -302.9962

Columns 50 through 56

-318.4706 -294.8956 -304.4663 -300.8919 -298.7732 -301.2474 -293.3188

Effects of High-Frequency Spectral Components

Aliased components at

$$f_{alias} = f_{sample} - f$$

$$f_{alias} = 1280 - 700 = 580\text{Hz}$$

$$\text{thus position in sequence} = 1 + N\frac{f_{alias}}{f_{sample}} = 1 + 512\frac{580}{1280} = 233$$

Columns 225 through 231

-296.8883 -292.8175 -295.8882 -286.7494 -300.3477 -284.4253 -282.7639

Columns 232 through 238

-273.9840 -6.0206 -274.2295 -284.4608 -283.5228 -297.6724 -291.7545

Columns 239 through 245

-299.1299 -305.8361 -295.1772 -295.1670 -300.2698 -293.6406 -304.2886

Columns 246 through 252

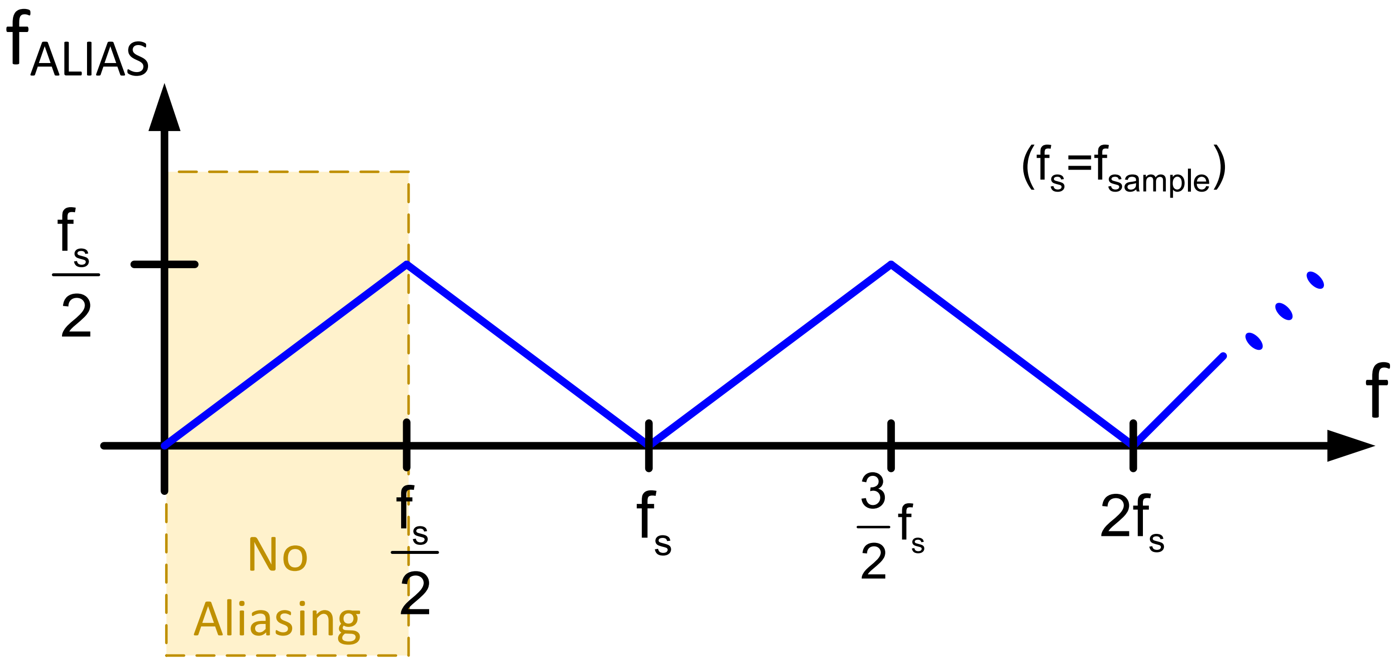
-302.0233 -306.6100 -297.7242 -305.4513 -300.4242 -298.1795 -299.0956

Alias frequency and Index Position

$$
f_{alias} = \begin{cases}
f \quad (\text{no alis}) & 0 < f < \dfrac{f_{sample}}{2} \\
f_{sample} - f & \dfrac{f_{sample}}{2} < f < f_{sample} \\
-f_{sample} + f & f_{sample} < f < \dfrac{3}{2} f_{sample} \\
2f_{sample} - f & \dfrac{3}{2} f_{sample} < f < 2f_{sample} \\
-2f_{sample} + f & 2f_{sample} < f < \dfrac{5}{2} f_{sample} \\
3f_{sample} - f & \dfrac{5}{2} f_{sample} < f < 3f_{sample} \quad \bullet\bullet\bullet
\end{cases}
$$

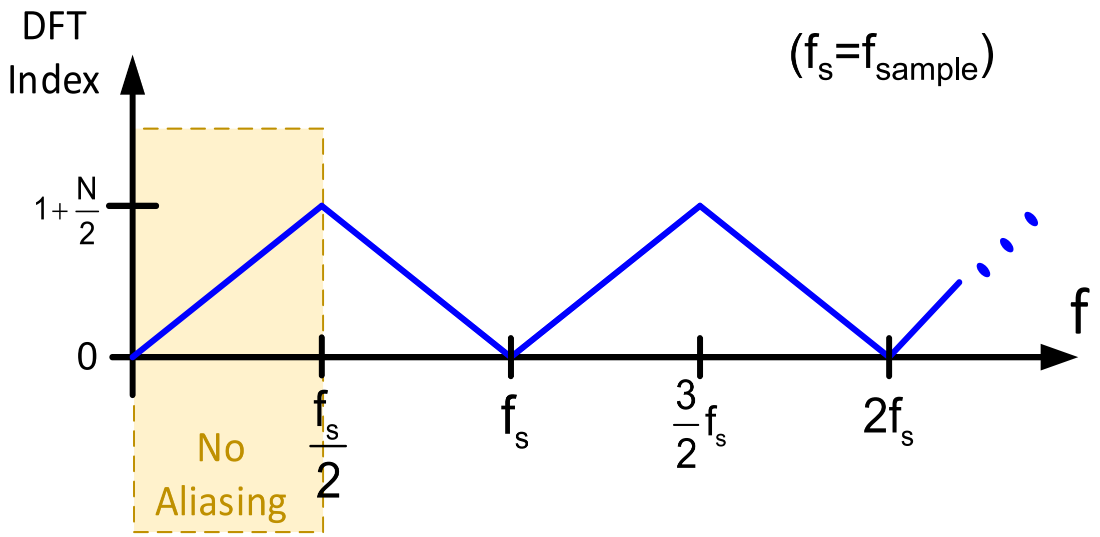
$$
\text{Index position in sequence} = 1 + N \frac{f_{alias}}{f_{sample}}
$$

Alias frequency and Index Position



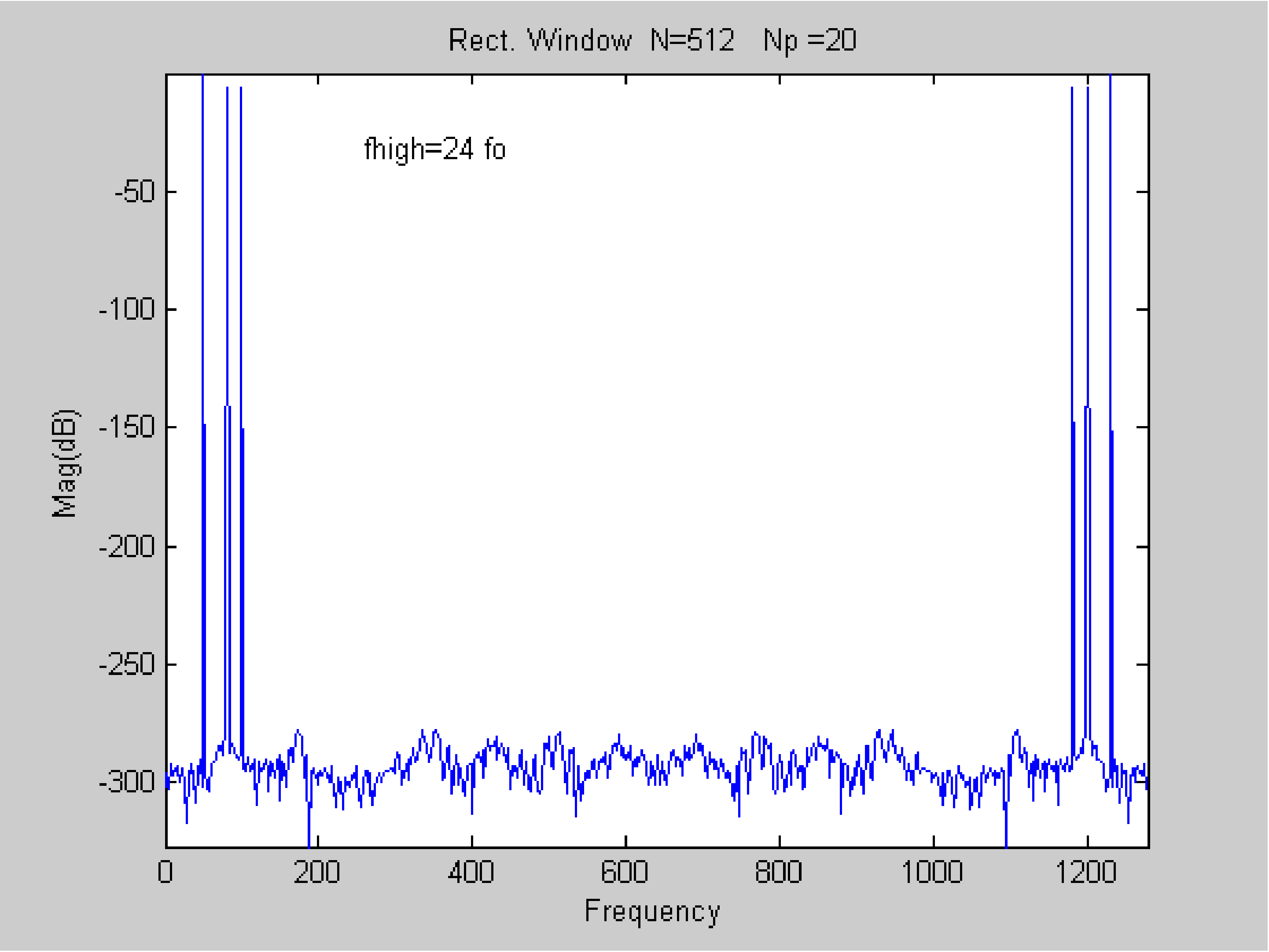
$$\text{Index position in sequence} = 1 + N \frac{f_{alias}}{f_{sample}}$$

Alias frequency and Index Position

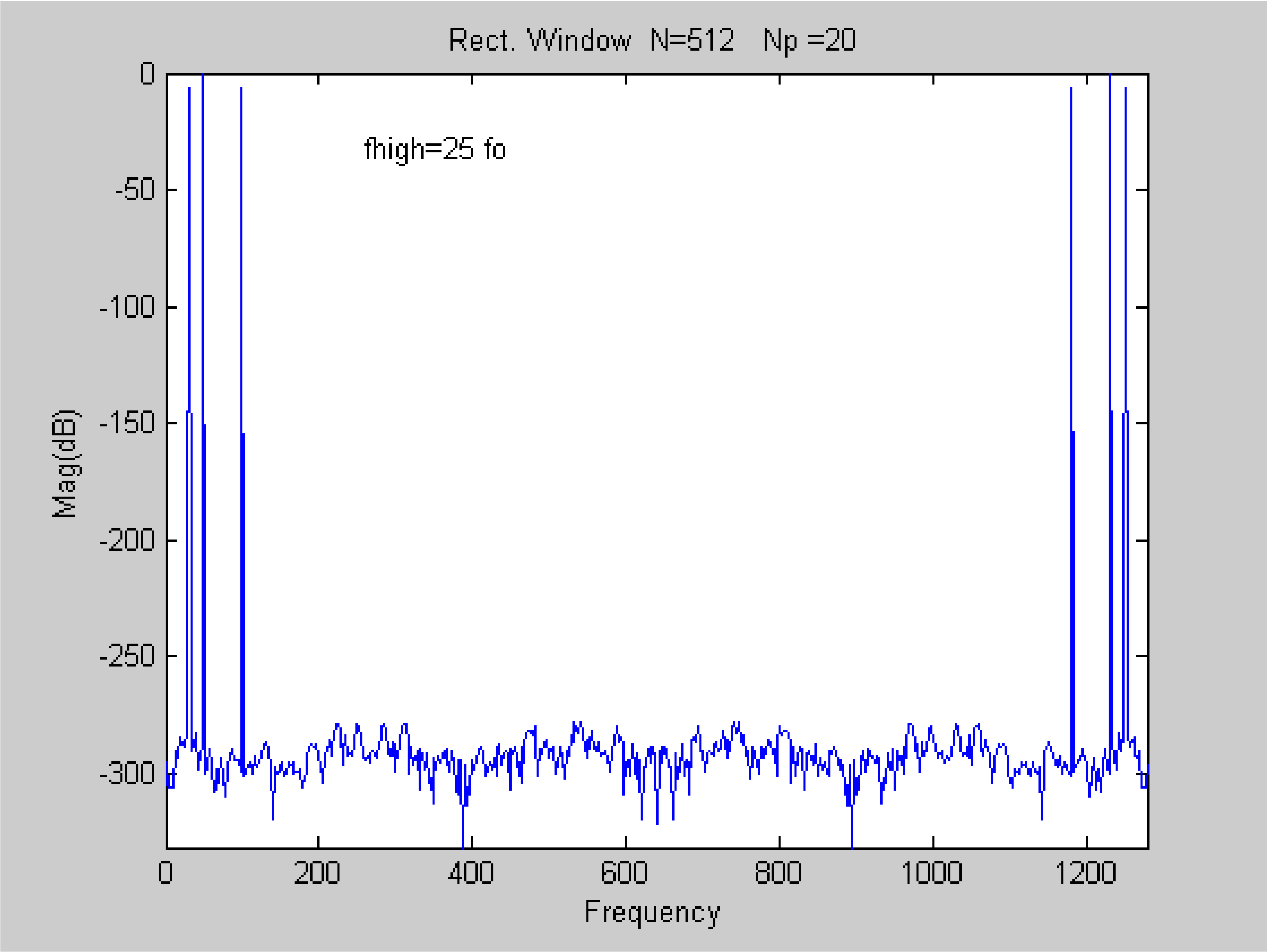


$$\text{Index position in sequence} = 1 + N\frac{f_{alias}}{f_{sample}}$$

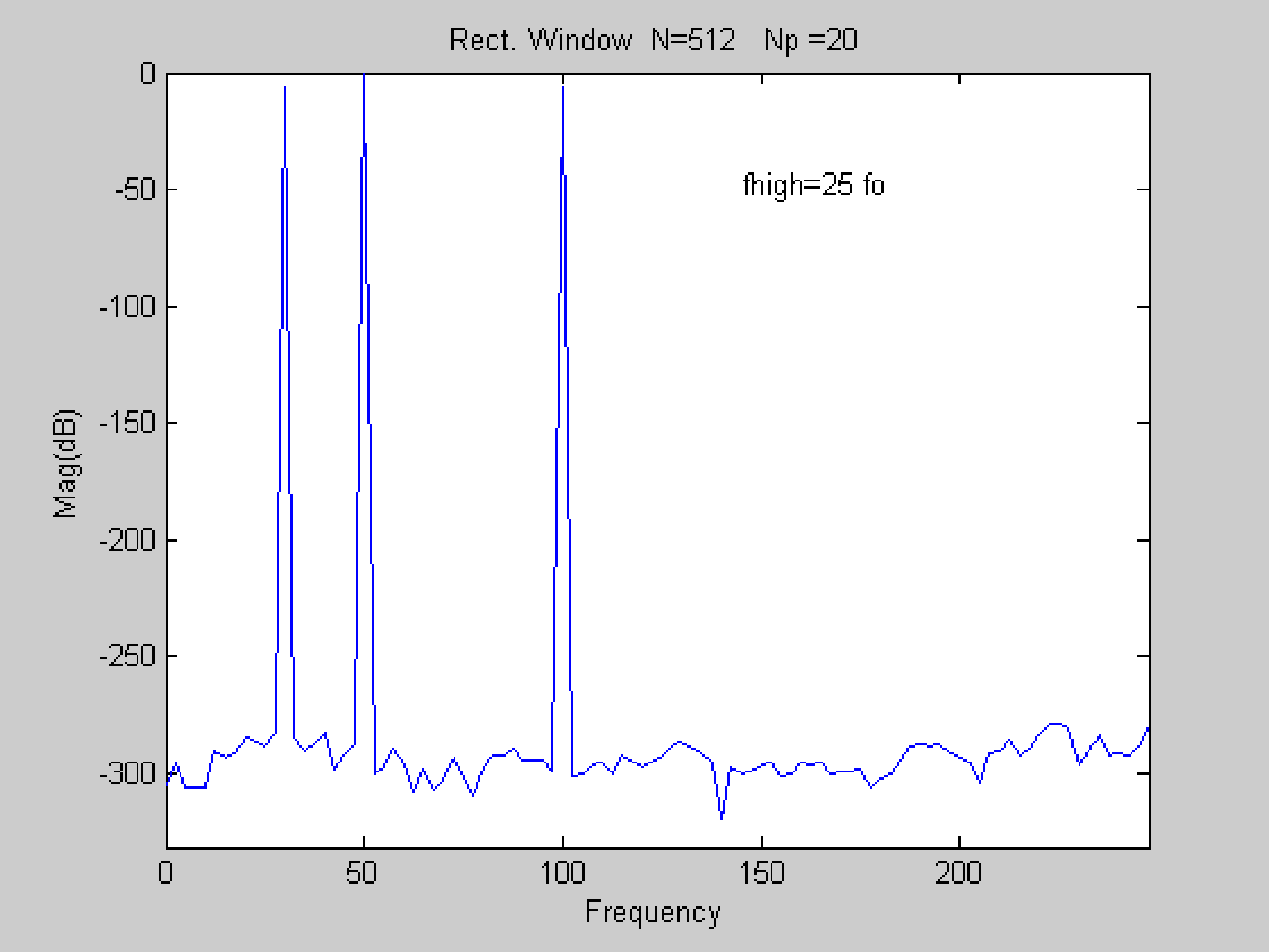
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

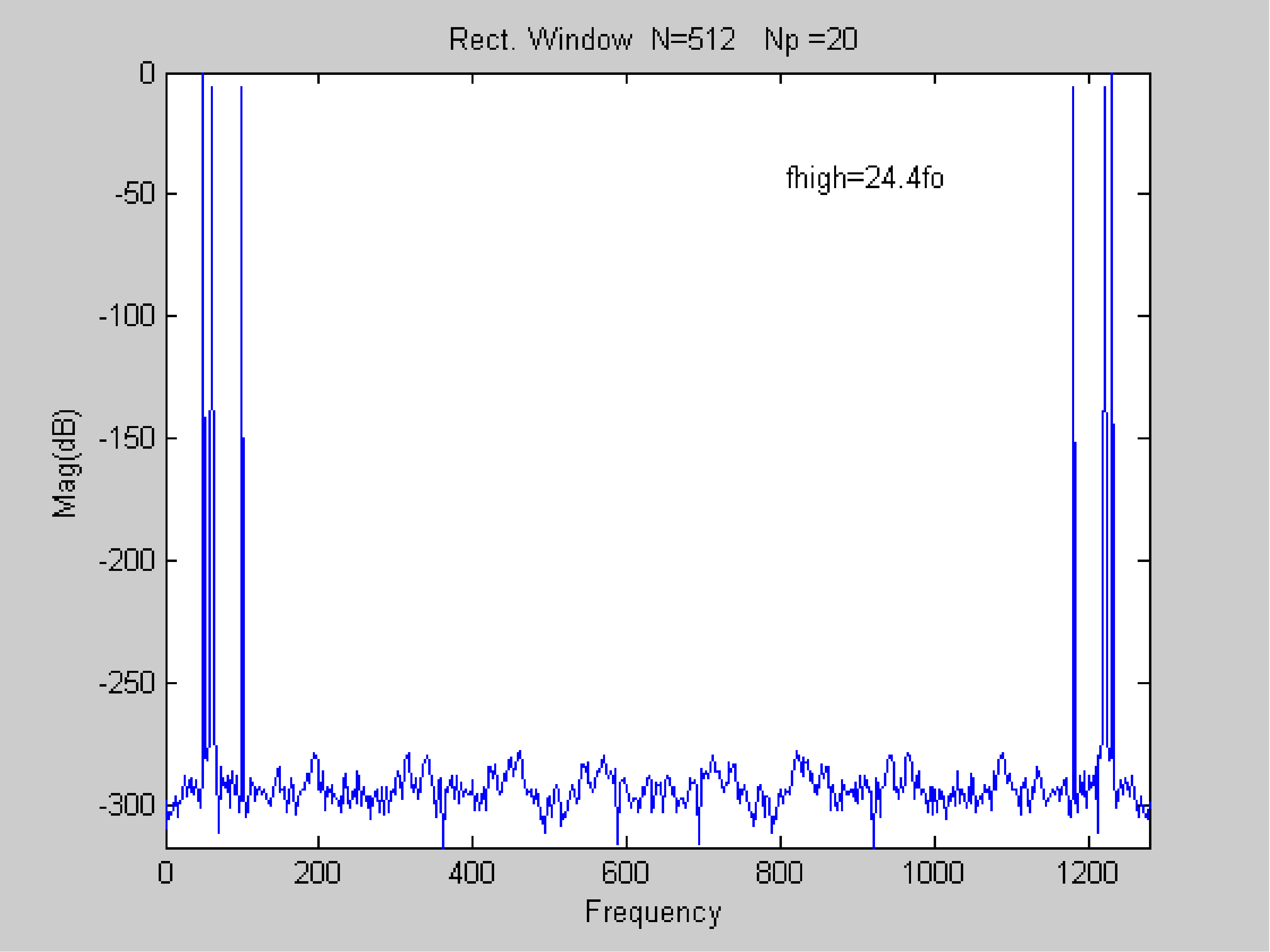


Effects of High-Frequency Spectral Components

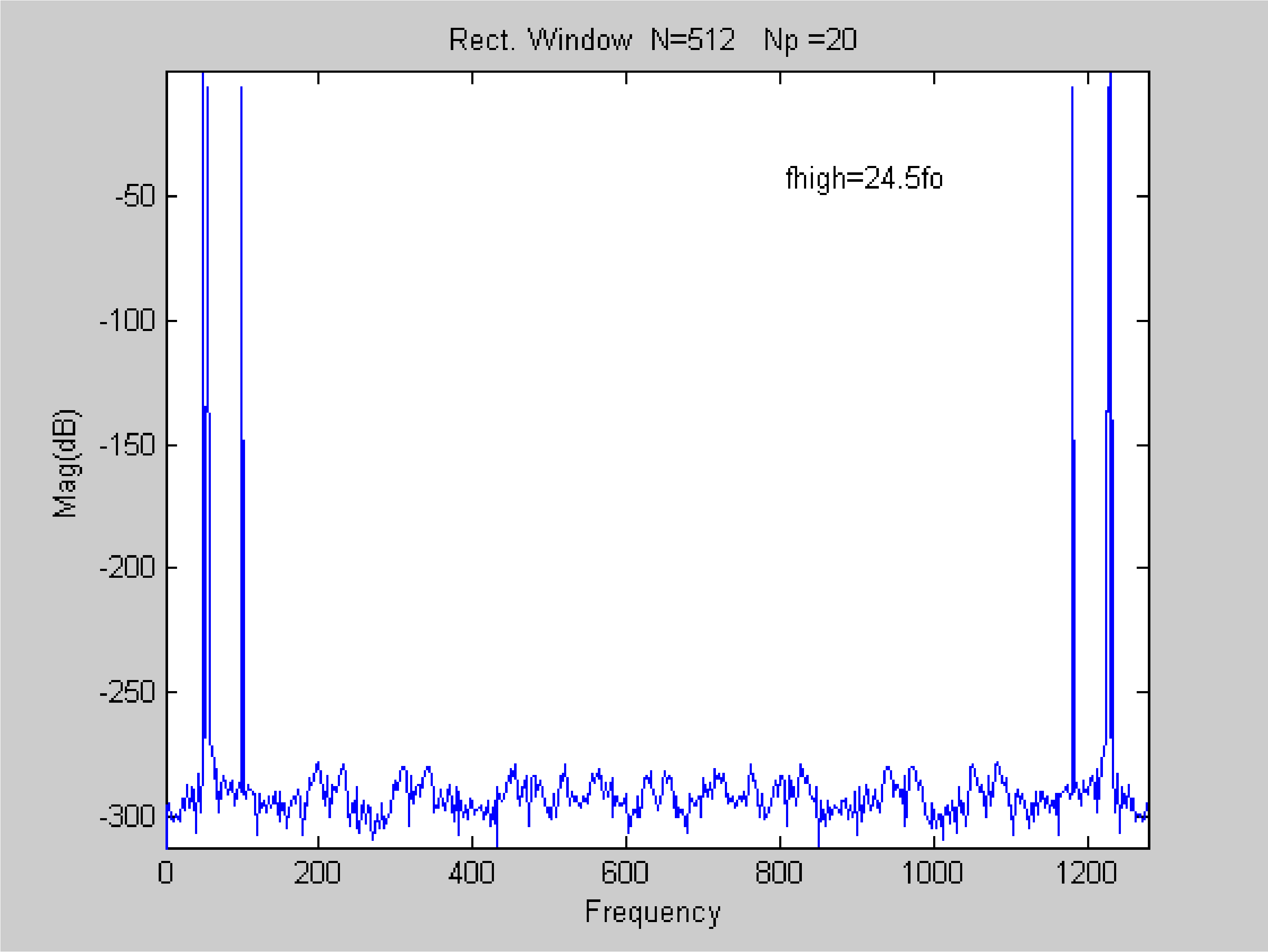


(zoomed in around fundamental)

Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

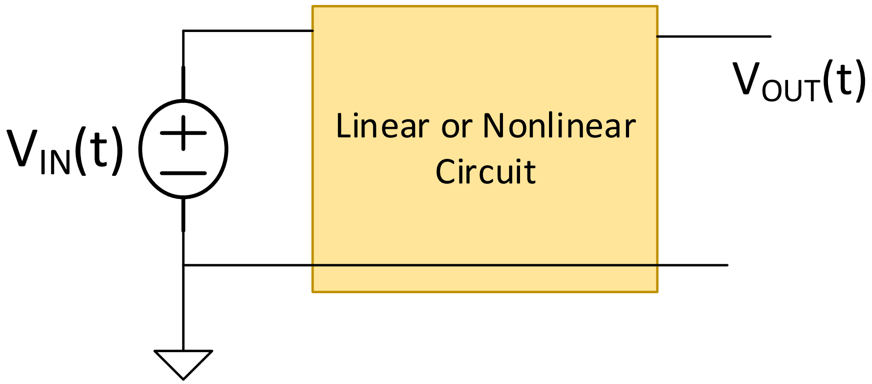


Observations

- Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied
- Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency
- More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics
- Important to avoid aliasing if the DFT is used for spectral characterization

Quirks with Circuit Simulators

The transient simulation results obtained with almost all circuit simulators are almost always wrong!

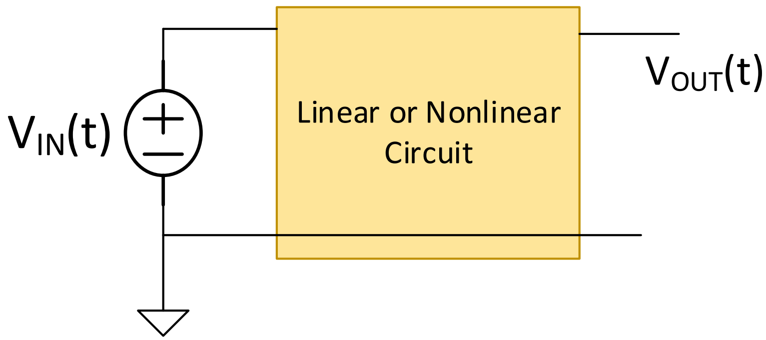


If $V_{IN}=V_m\sin(\omega t)$, the probability that any output for any linear or nonlinear circuit is the correct value is likely 0

The suppliers of all commercial circuit simulators know that their simulators almost never provide the correct solution for transient analysis in even simple circuits

But the solutions provided are often very good approximations to the actual solution

Quirks with Circuit Simulators



Obtaining an exact result for a transient simulation is an extremely challenging problem and even obtaining very good results have required major efforts by the CAD community for many decades

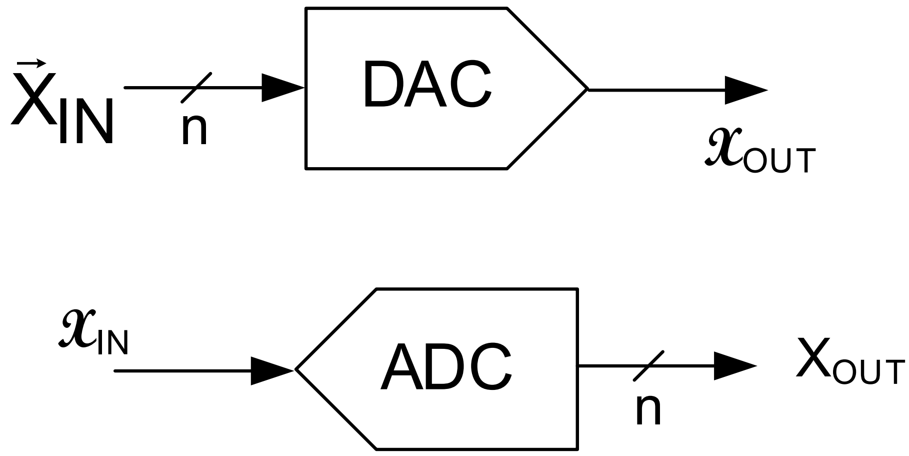
It is highly unlikely that the goal of any CAD tool vendor is to provide the exact transient solution

So what is likely the goal of a CAD tool vendor when developing tools to provide the transient response of a circuit?

Conjecture: To provide solutions that are good enough to convince customers to spend money to use the simulator !

Is this a good goal that well-serves the customer?

Quirks with Circuit Simulators

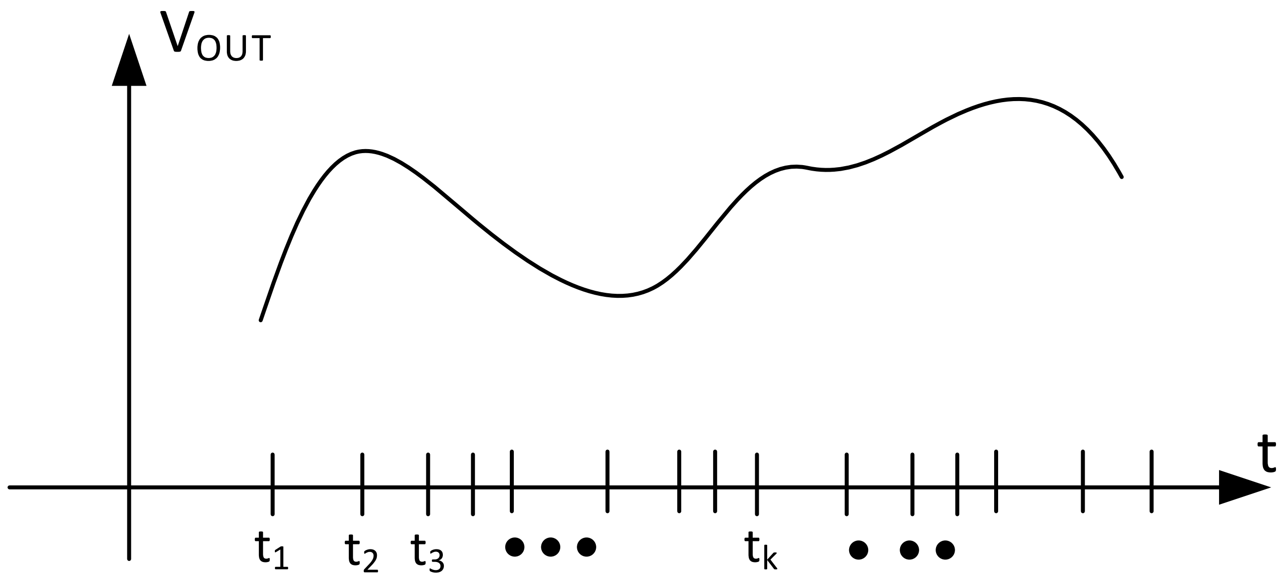


There are numerous quirks that become issues when simulating ADCs of DACs , particularly when the specifications are demanding

Will discuss one of these quirks associated with spectral characterization today

Quirks with Circuit Simulators

Time steps in transient simulations



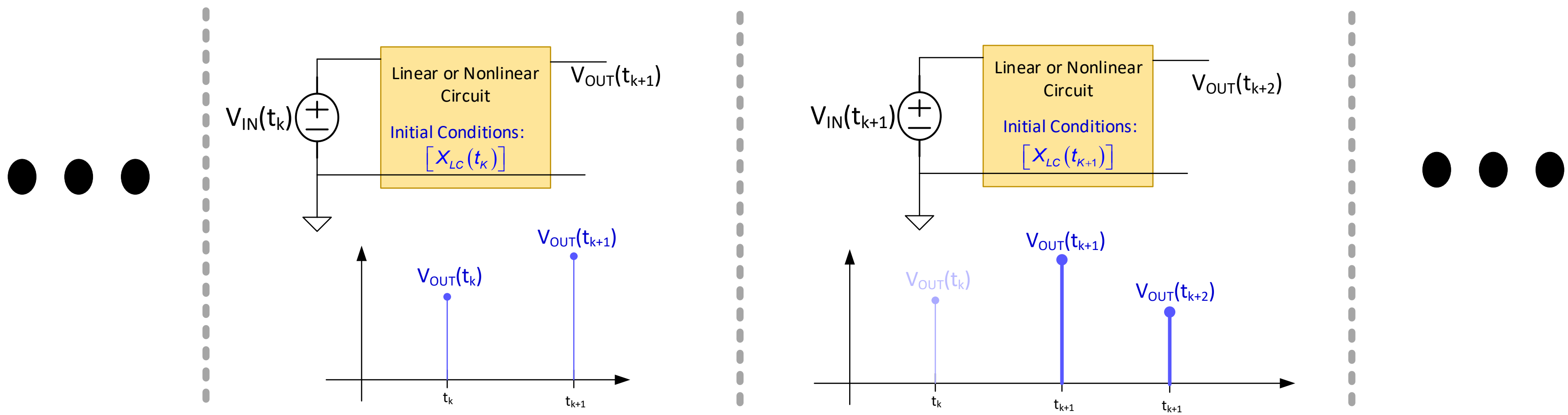
Transient analysis in SPICE involves breaking the simulation time interval into short sub-intervals of length $\widehat{t}_k = t_{k+1} - t_k$

The solution involves using the calculated output at time t_k as the input at time t_{k+1}

The time steps are intentionally not uniform to provide reasonable tradeoffs between simulation time and accuracy

Quirks with Circuit Simulators

Transient Simulation Approach in Simulators

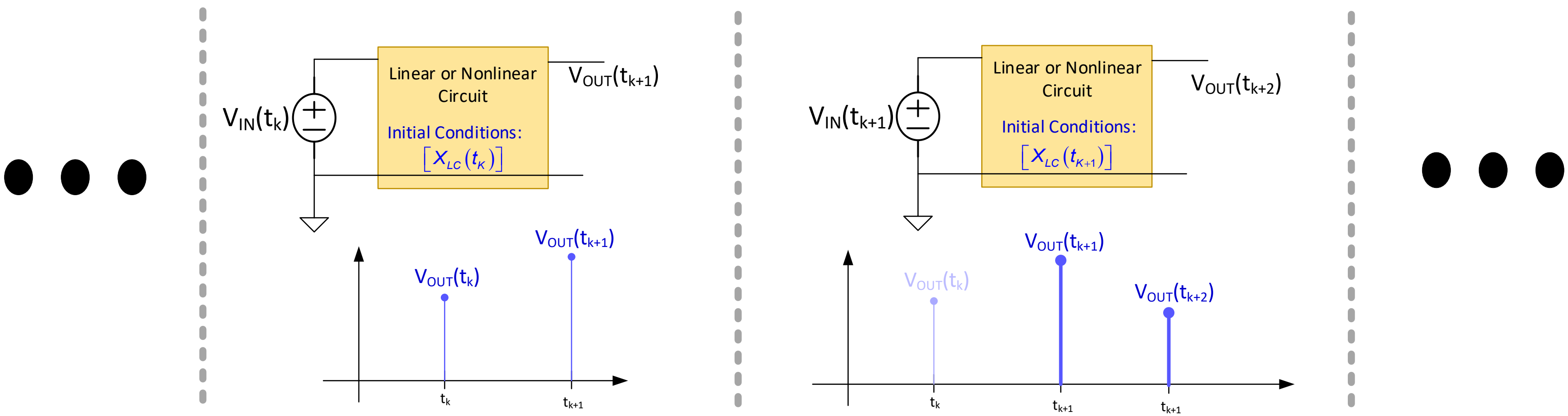


$\left[X_{LC}(t)\right]$ is the vector of the instantaneous value of all capacitor voltages and all inductor currents

- Transient simulations are decomposed into a sequence of individual problems where the output of all energy storage elements at step k serves as the initial condition (not initial guess) of all energy storage elements at step k+1
- Simulator attempts to conserve charge in each simulation step. This is a key concept that must be incorporated for consistency in any circuit simulator when Ls and Cs are included in schematic.

Quirks with Circuit Simulators

Transient Simulation Approach in Simulators



Simulator attempts to conserve charge in each simulation step (KCL and KVL)

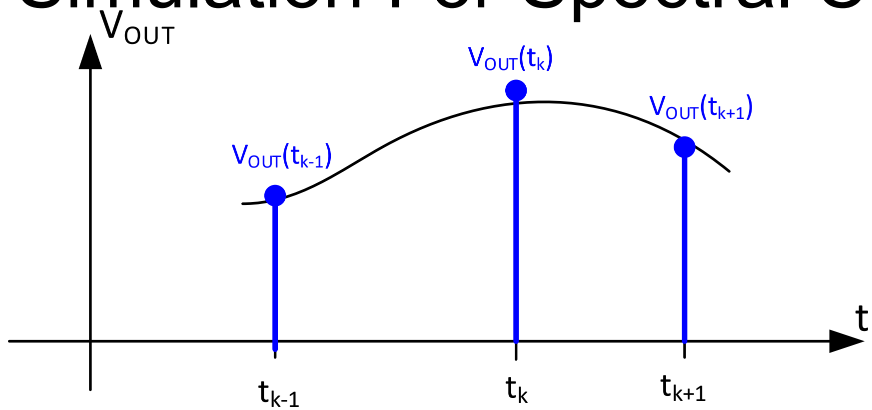
Basic physical principle that is integral to any transient simulation of circuits with energy storage elements: CHARGE IS CONSERVED !!

Is charge conserved in a circuit simulator when doing transient simulations?

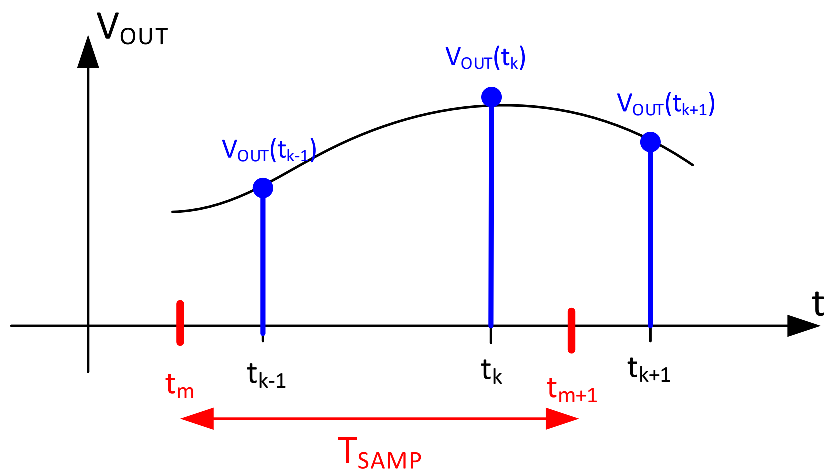
No !   It is only conserved locally (in individual time intervals) so major divergence can occur in some extreme situations due to accumulative round off effects

Quirks with Circuit Simulators

Transient Simulation For Spectral Characterization



Normal time-stepping algorithm is used to obtain transient response

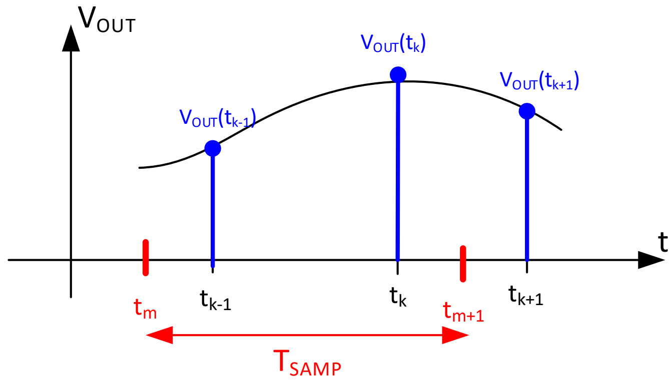


DFT requires output at precisely uniformly spaced predetermined points, t_m,t_{m+1}…

These points are almost never coincident with the time-stepping points !

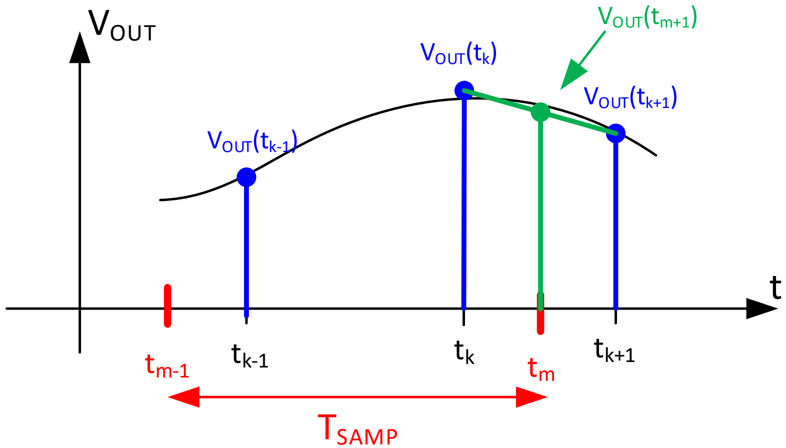
Quirks with Circuit Simulators

Transient Simulation For Spectral Characterization



So how does the simulator generate outputs at the required time points ?

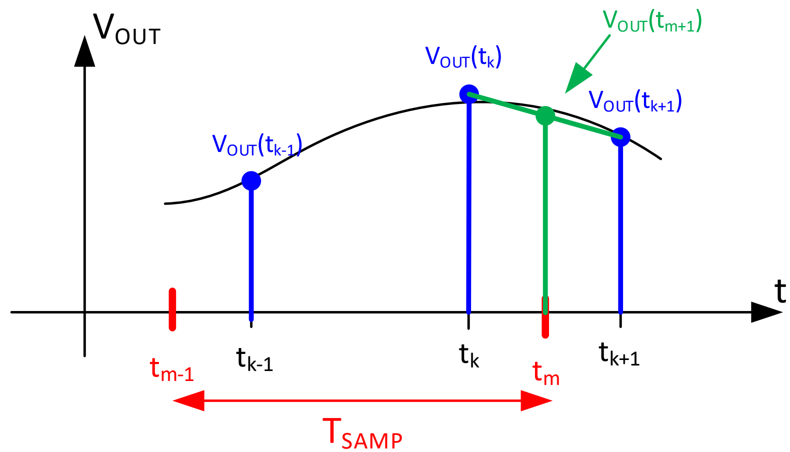
It interpolates ! Exact interpolation algorithm may vary from vendor to vendor



For linear interpolation
$$V_{OUT}(t_m) = V_{OUT}(t_k) + \left[V_{OUT}(t_{k+1}) - V_{OUT}(t_k)\right]\frac{t_{m+1} - t_k}{t_{k+1} - t_k}$$

Quirks with Circuit Simulators

Transient Simulation For Spectral Characterization



For linear interpolation

$$V_{OUT}(t_m) = V_{OUT}(t_k) + \left[V_{OUT}(t_{k+1}) - V_{OUT}(t_k)\right]\frac{t_{m+1} - t_k}{t_{k+1} - t_k}$$

What errors are introduced in determining $V_{OUT}(t_m)$?

- Errors in calculating $V_{OUT}(t_k)$

  (most problematic for long simulations with multiple energy storage elements)

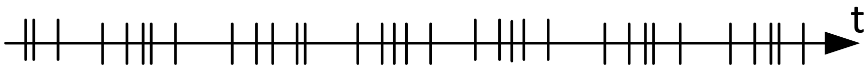
  (may not be particularly problematic for spectral characterization since N_P often not too long)

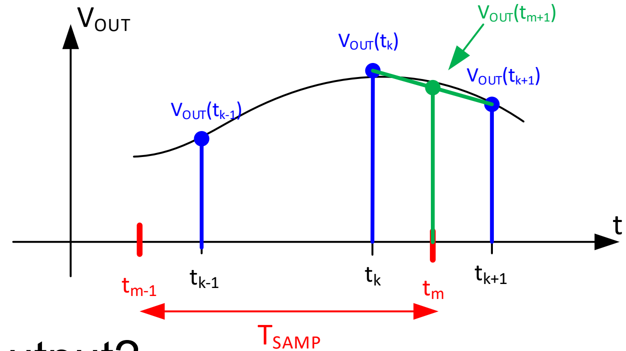
  (must simulate long enough for natural response to die out if ALL initial conditions are not correctly set)

- Errors associated with interpolation $V_{OUT}(t_k)$

Quirks with Circuit Simulators

If interpolation is a problem, what can be done about it

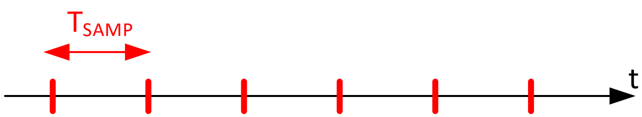




For linear interpolation

$$V_{OUT}(t_m) = V_{OUT}(t_k) + \left[V_{OUT}(t_{k+1}) - V_{OUT}(t_k)\right]\frac{t_{m+1}-t_k}{t_{k+1}-t_k}$$

- specify the sample points where you want output?

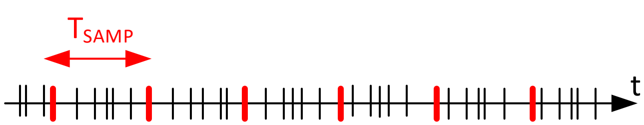


No – it will ignore such request

- specify a very small maximum step size

May help but maybe not enough, long sim times, possibly convergence problems

- modify time stepping algorithm to include desired sample points



May help a lot ! Use Strobe Period function if available in simulator

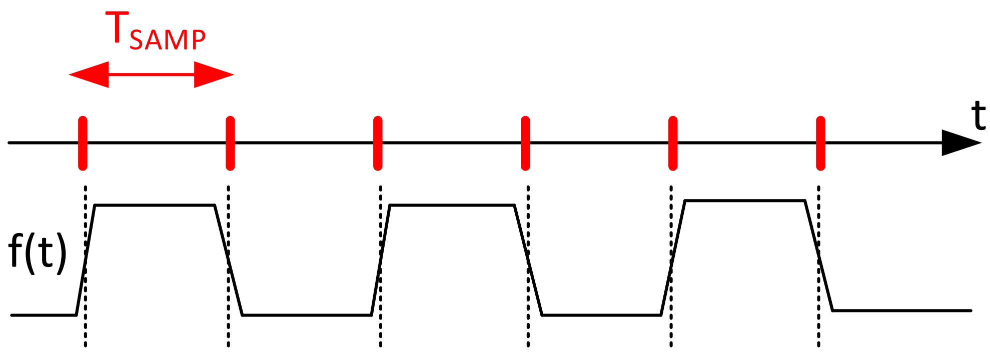
- Add noninteracting waveform with steep slope at desired transition points

May also help a lot ! Forces some time stepping algorithms to take sample near sample points

Quirks with Circuit Simulators

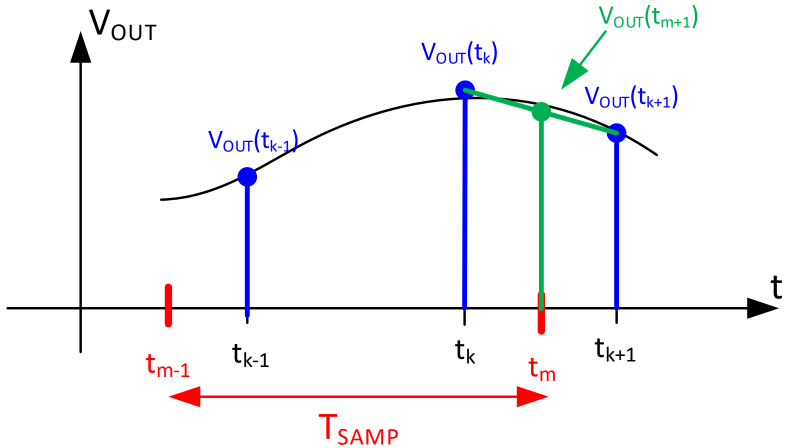
If interpolation is a problem, what can be done about it

- Add noninteracting waveform with steep slope at desired transition points



Quirks with Circuit Simulators

Transient Simulation For Spectral Characterization



Errors associated with interpolation $V_{OUT}(t_k)$

Will consider example using Spectre

Do not know what the interpolation algorithm is

Will not include any energy storage elements

Spectre Limitations in Spectral Analysis

Thanks to Xilu Wang for simulation results

- Normal Transient Analysis
- Strobe Period Timing
- Coherent Sampling

Simulation Conditions

$V(t)=\sin(2*\pi*50t)$

11 periods

Coherent Sampling

Number of Samples:

- 512
- 4096

Type of Samples:

- Standard Sweep
- Strobe Period Sweep

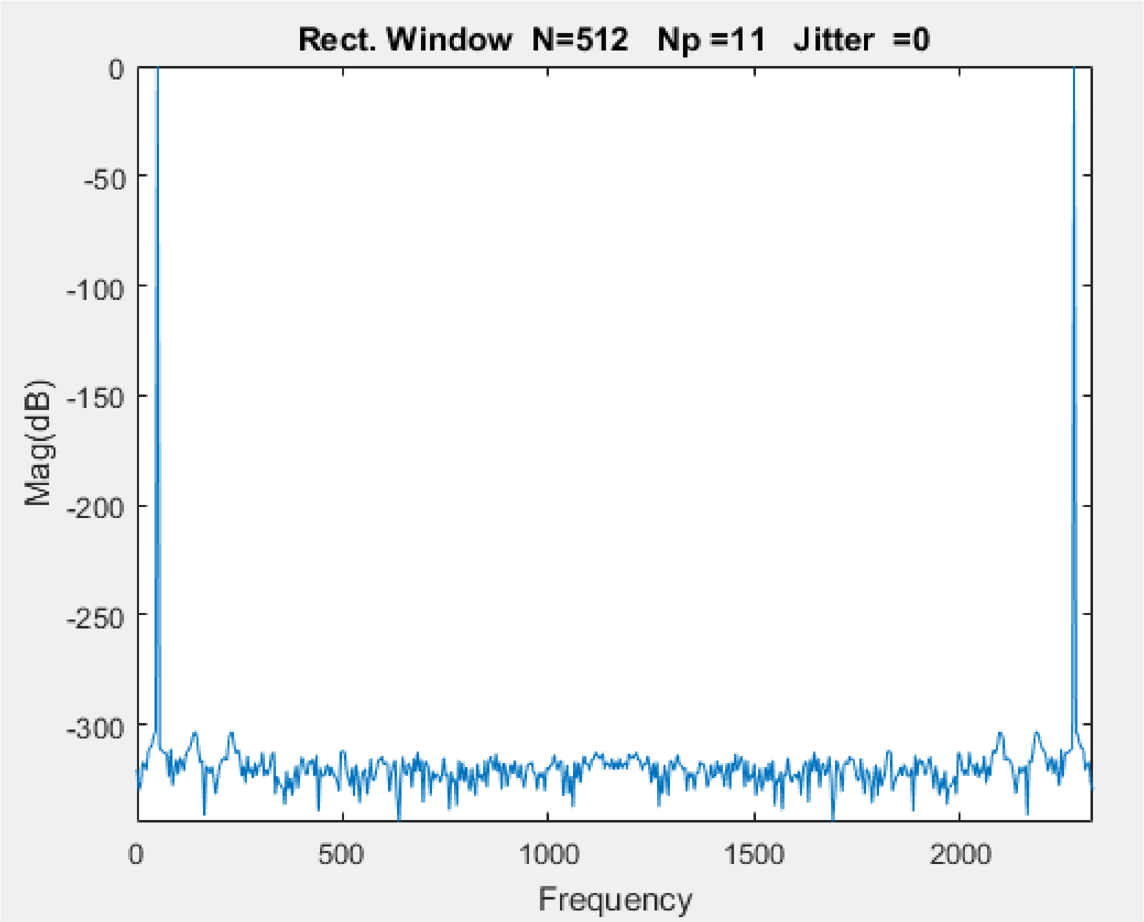
512 Samples with Standard Sweep

$V(t)=\sin(2*\pi*50t)$

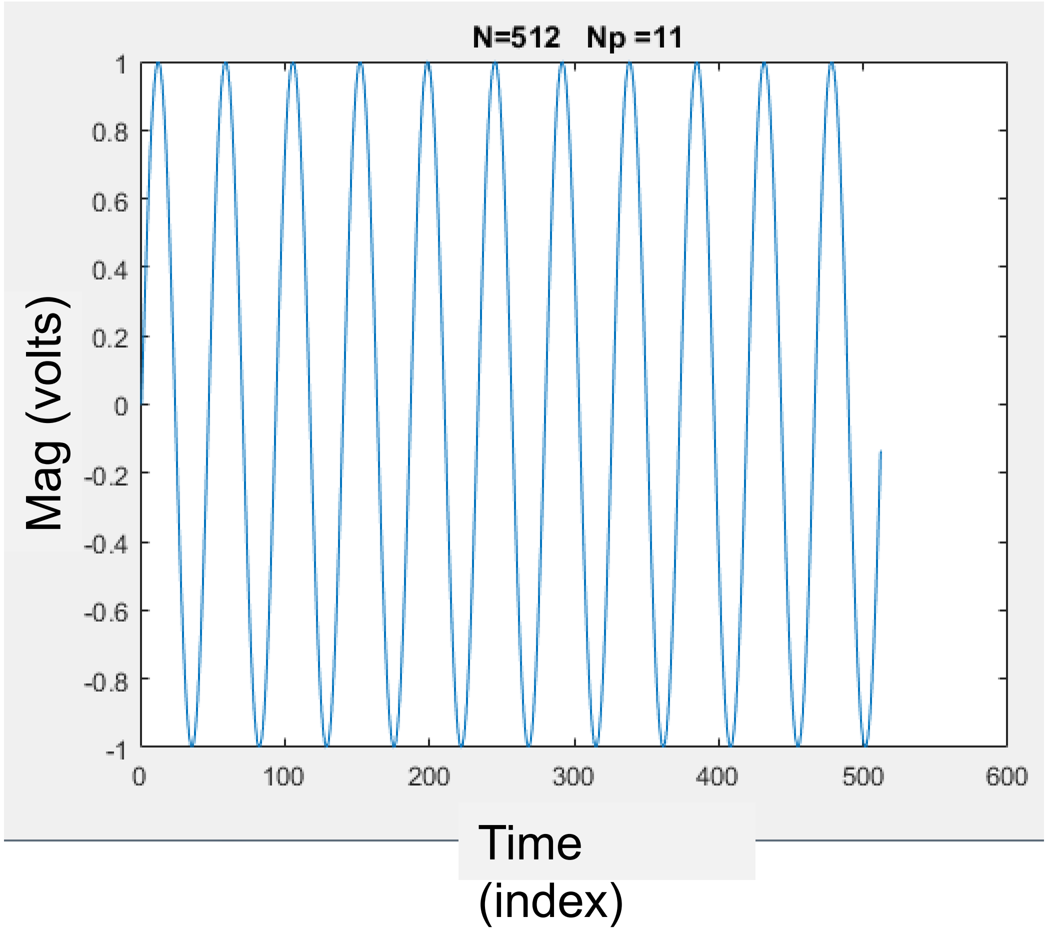
11 periods

Coherent Sampling

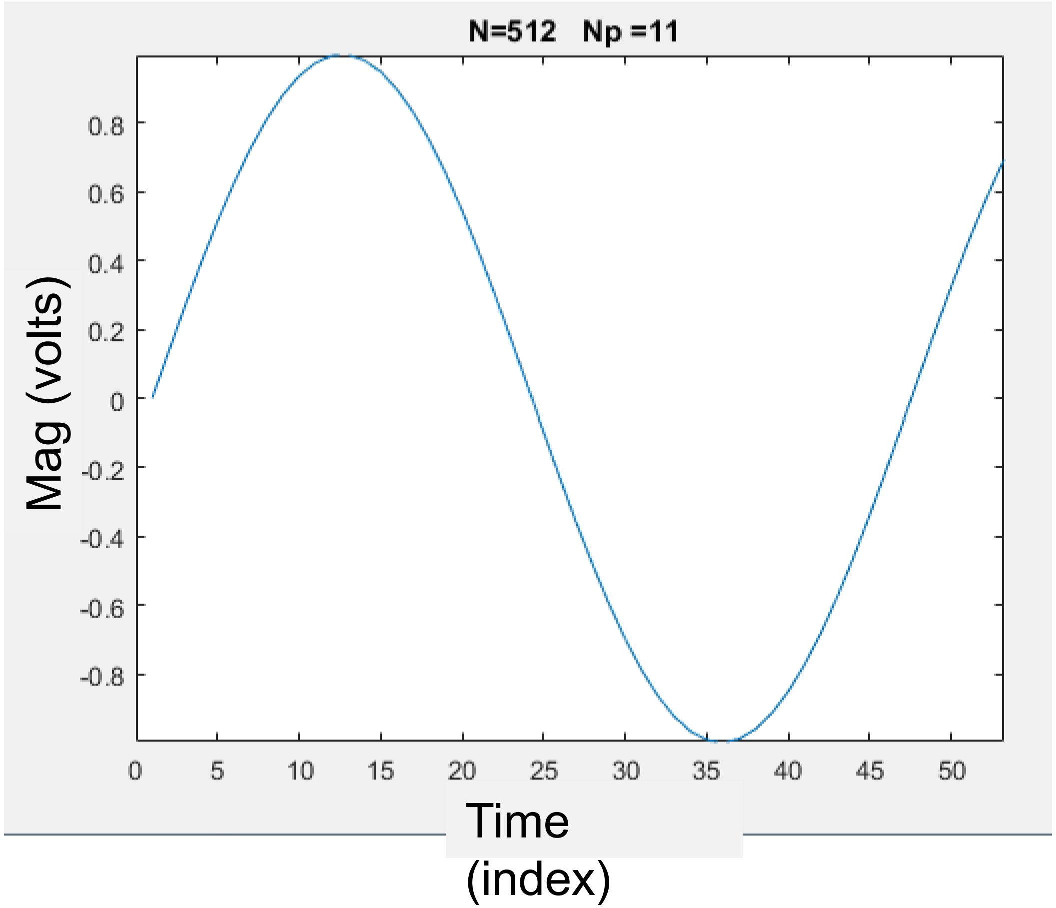
For reference: Results obtained with MatLab for N=512



512 Samples with Standard Sweep



512 Samples with Standard Sweep



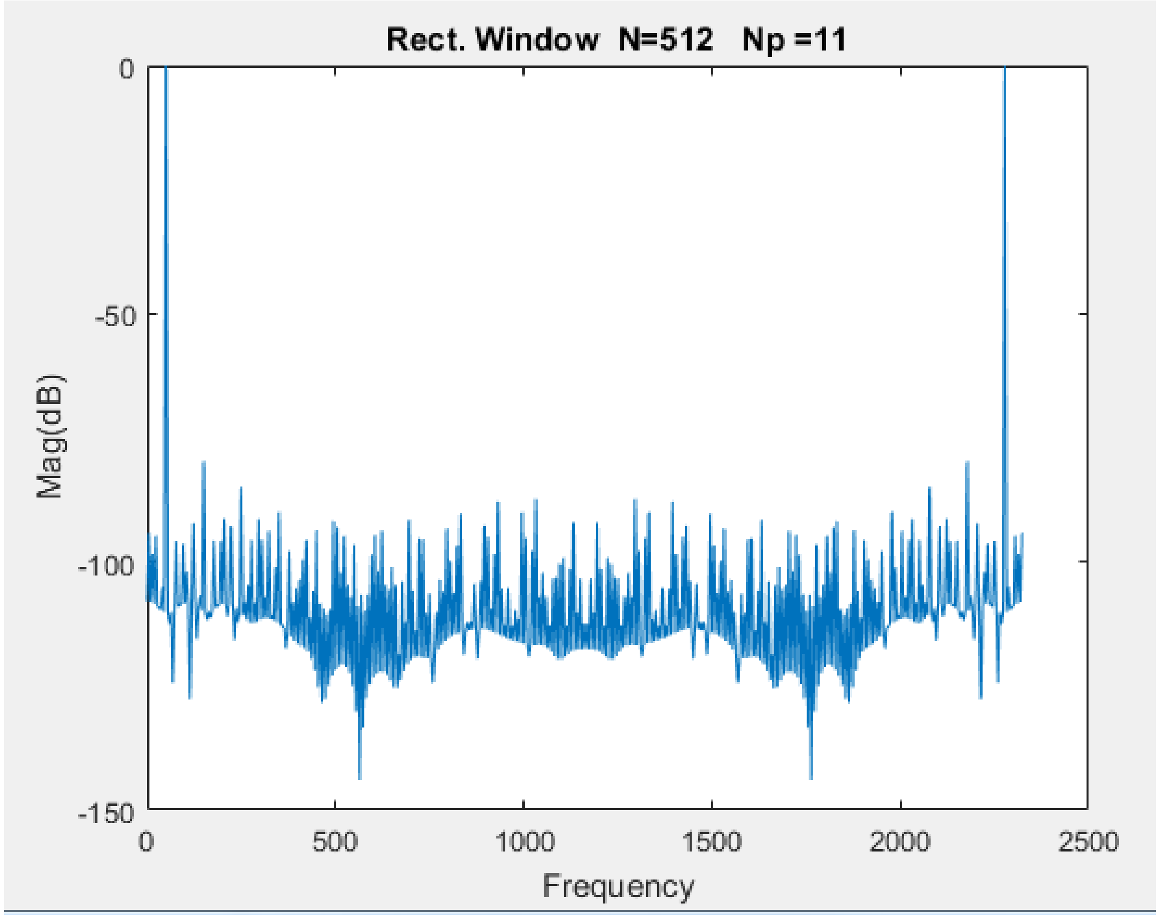
512 Samples with Standard Sweep

```
Pyyt1 =

  Columns 1 through 12

 -107.8981  -94.1331 -108.0317  -98.4157 -108.4194  -94.7571 -108.9965 -109.0037 -109.6020 -105.0095 -110.0367   -0.0025

  Columns 13 through 24

 -110.1847 -112.5300 -110.0274 -124.3662 -109.6023  -95.7592 -109.0660 -108.3765 -108.6183  -96.3597 -108.3281 -101.9470

  Columns 25 through 36

 -108.2245 -127.5677 -108.3644  -92.2404 -108.7400 -115.5801 -109.2418 -106.8007 -109.7118  -79.7378 -109.9937 -111.7277

  Columns 37 through 48

 -109.9798 -111.2019 -109.6609  -95.7148 -109.1668 -108.6139 -108.7231  -95.8926 -108.4382  -91.2831 -108.3050 -110.9404
```

512 Samples with Standard Sweep

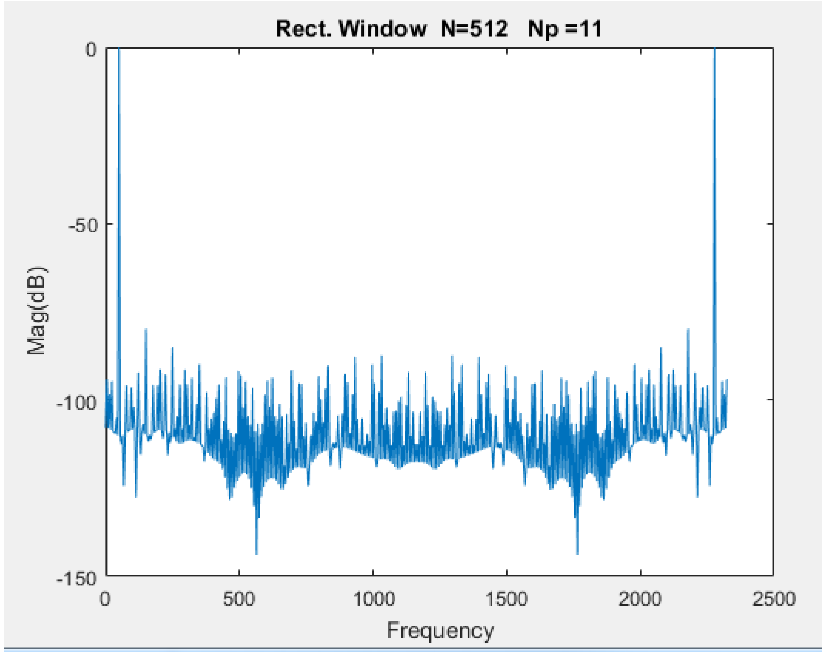
```
 Columns 49 through 60

-108.3530  -92.7636 -108.6349 -115.7449 -109.1516 -112.1106 -109.8730  -84.9671 -110.7375 -108.1262 -111.5945 -104.6449

 Columns 61 through 72

-112.1962  -95.5508 -112.3197 -108.1874 -112.0368  -91.3543 -111.6457  -95.5037 -111.3708 -107.0414 -111.3212  -93.7515

 Columns 73 through 84

-111.5009 -110.5545 -111.8098 -101.0709 -112.1458  -89.9120 -112.5548 -112.9274 -113.1741 -117.4284 -114.0466  -97.7820

 Columns 85 through 96

-115.0135 -108.1029 -115.8627 -105.3494 -116.4956 -103.5153 -116.9316  -99.5311 -117.3540  -95.6142 -118.0994 -111.2058

 Columns 97 through 108

-119.5251 -105.8620 -121.8988  -93.6210 -125.2388 -108.3341 -128.2779 -109.3118 -127.5511 -110.5460 -124.9168 -109.4982
```

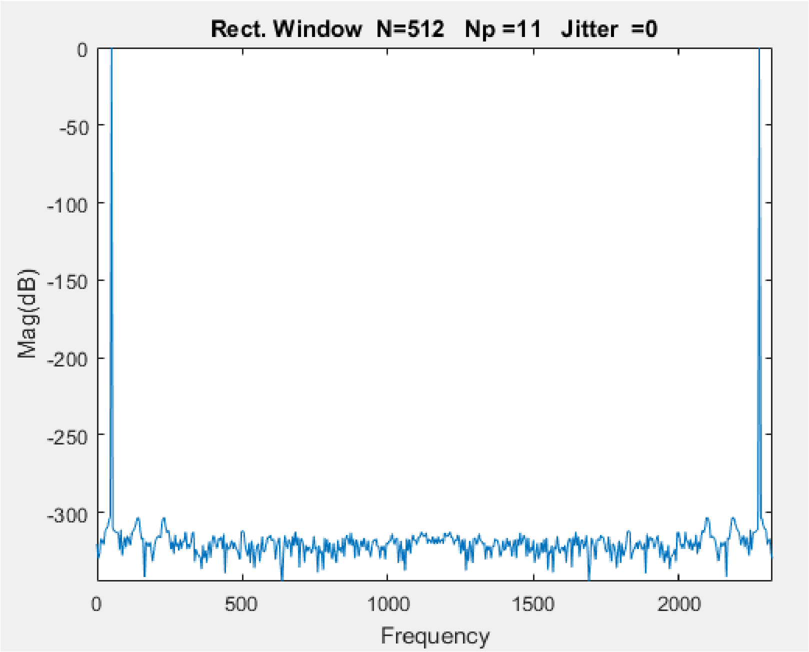
512 Samples with Standard Sweep



- Note dramatic increase in noise floor
- Note what appear to be some harmonic terms extending above noise floor

MatLab comparison: 512 Samples with Standard Sweep

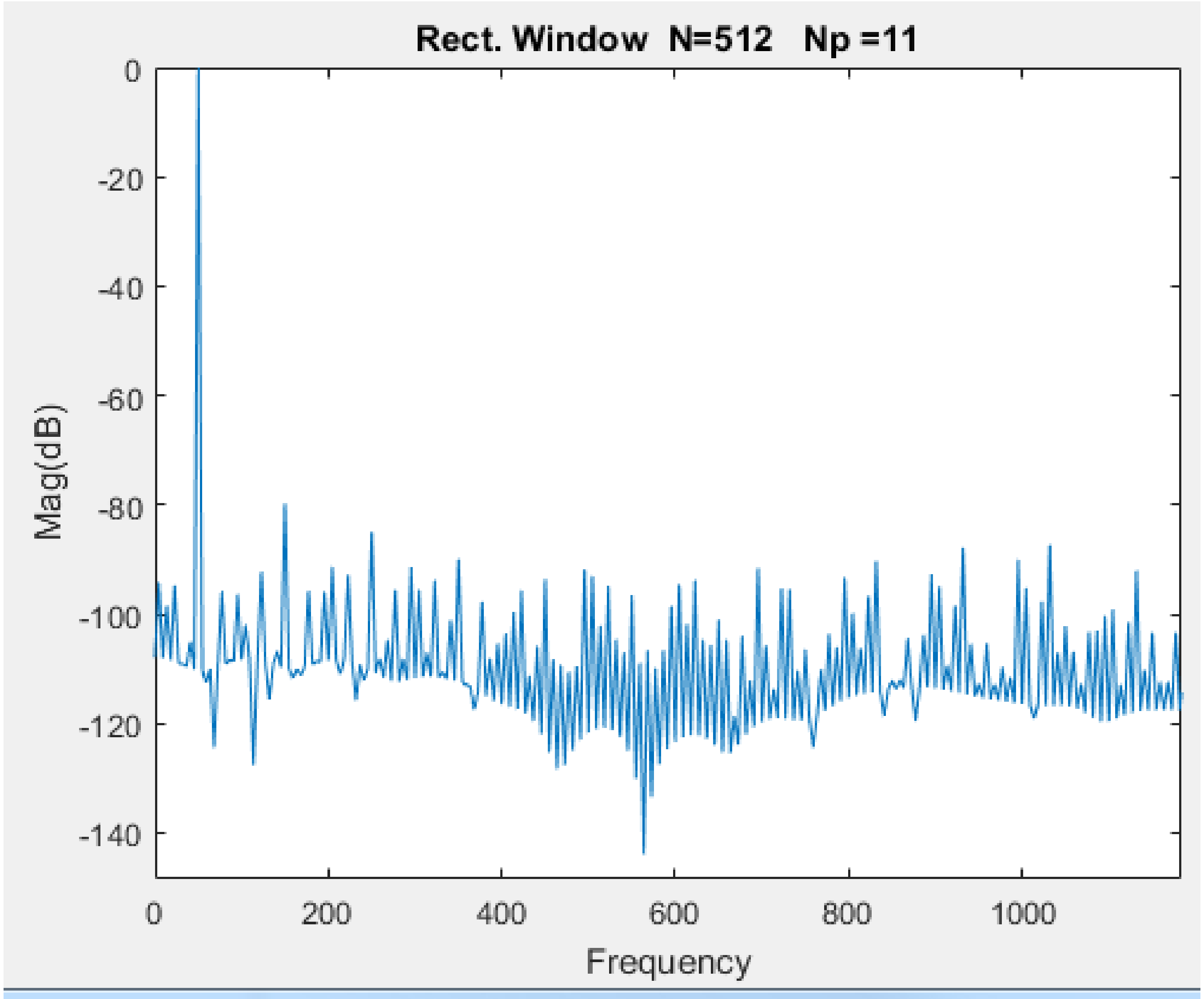




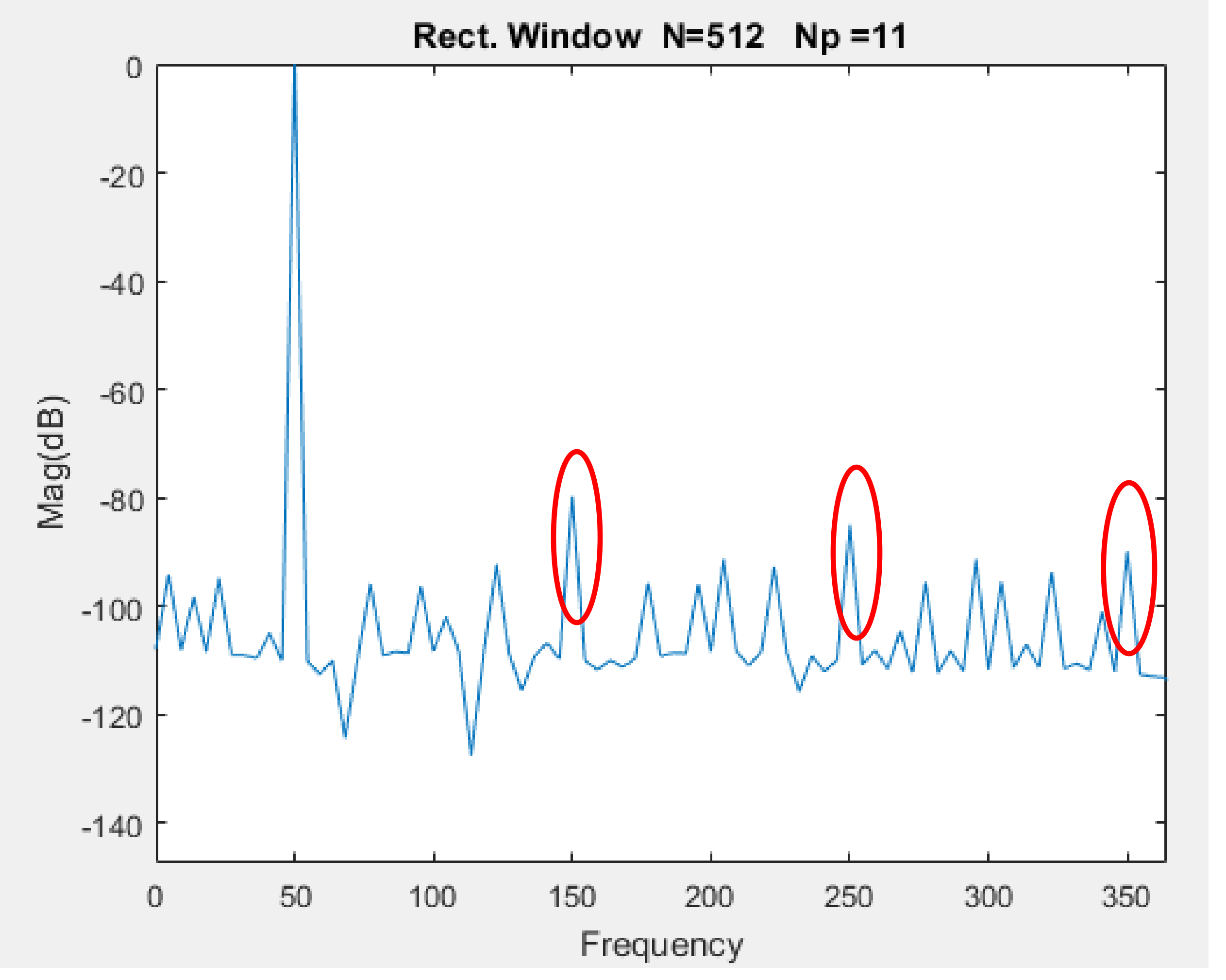
Spectre Results

MatLab Results

512 Samples with Standard Sweep

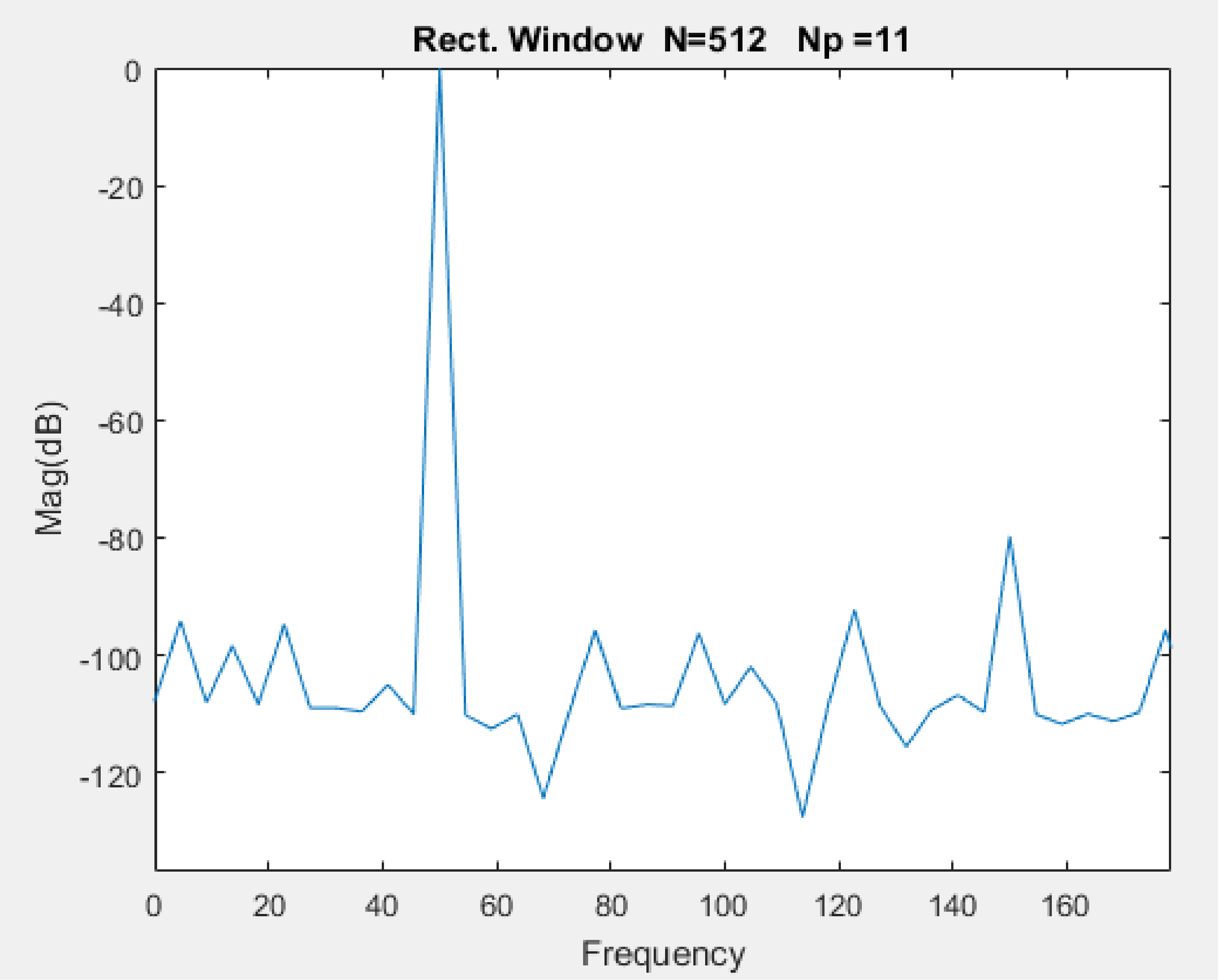


512 Samples with Standard Sweep



Note presence of odd harmonics in spectrum

512 Samples with Standard Sweep



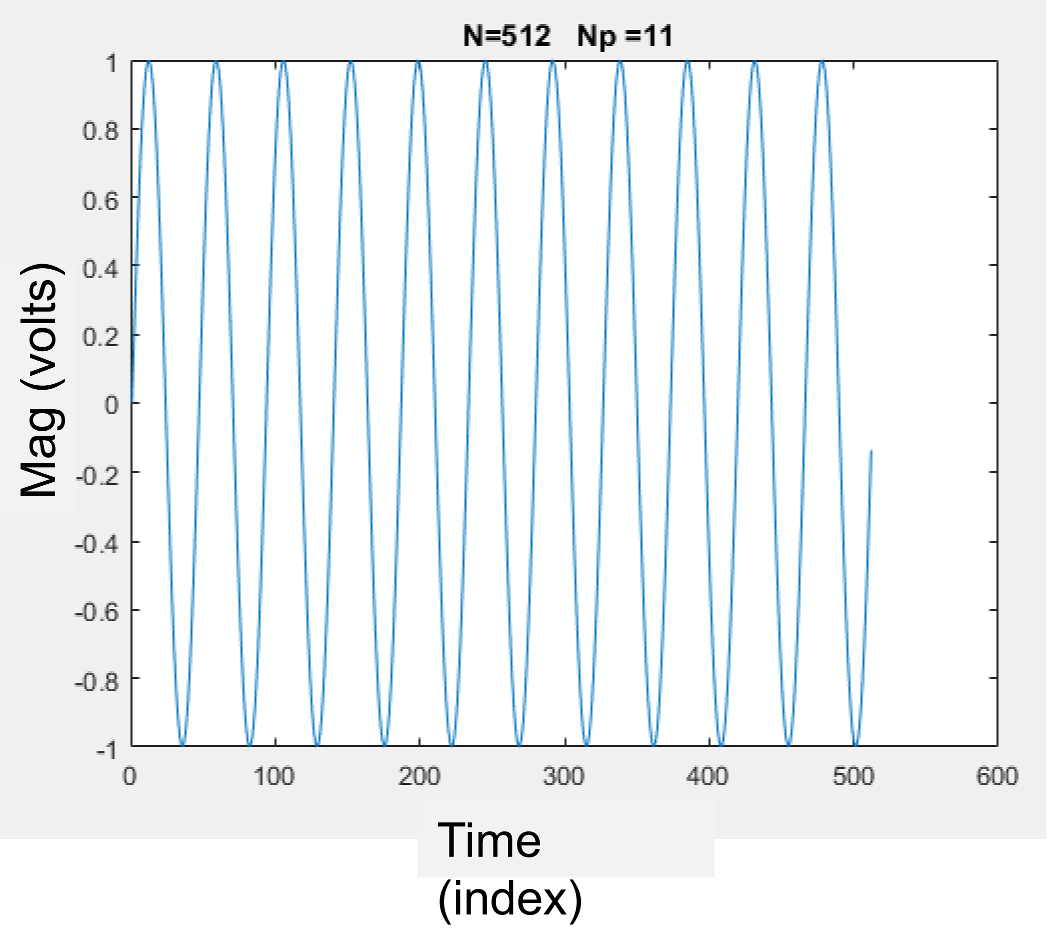
512 Samples with Strobe Period Sweep

$V(t)=\sin(2*\pi*50t)$

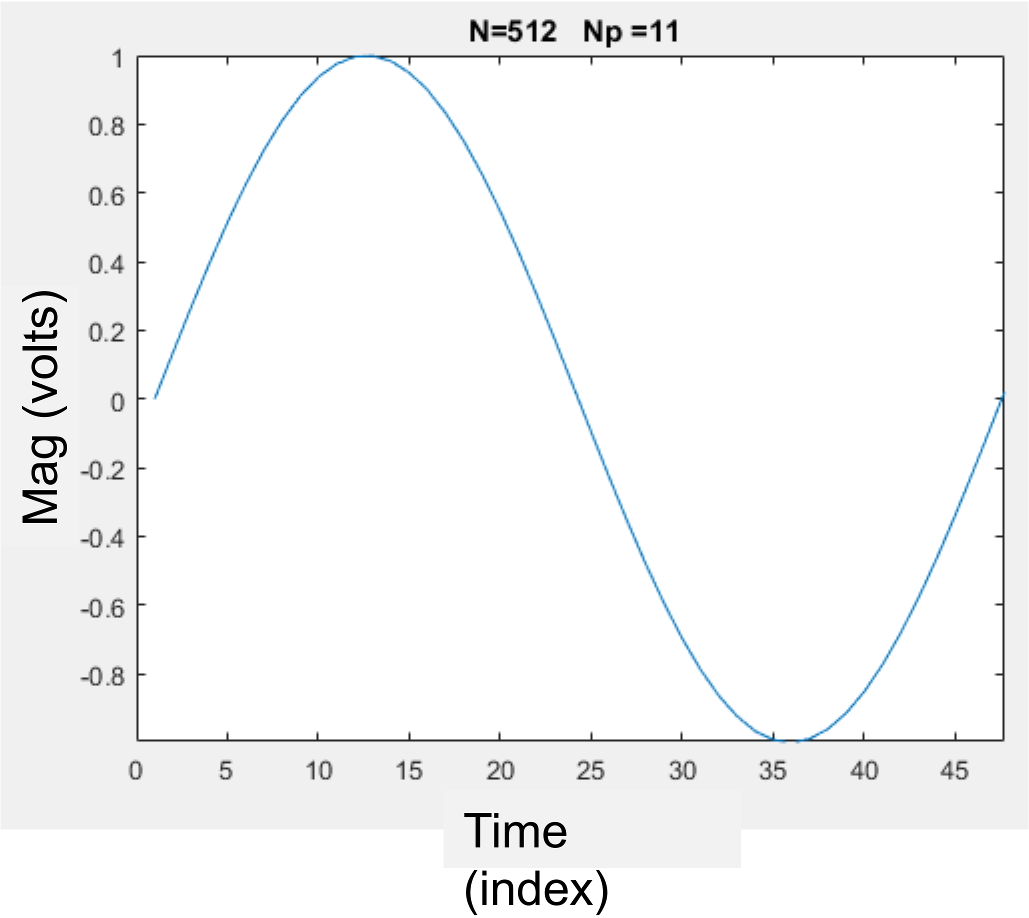
11 periods

Coherent Sampling

512 Samples with Strobe Period



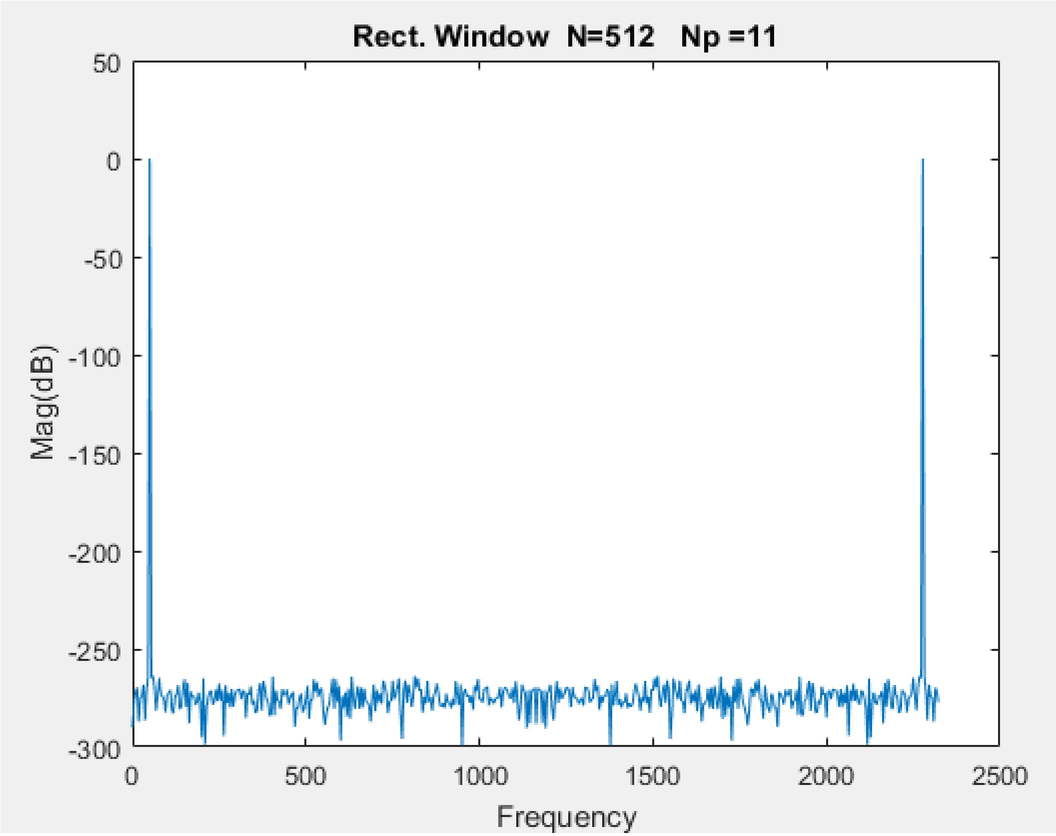
512 Samples with Strobe Period



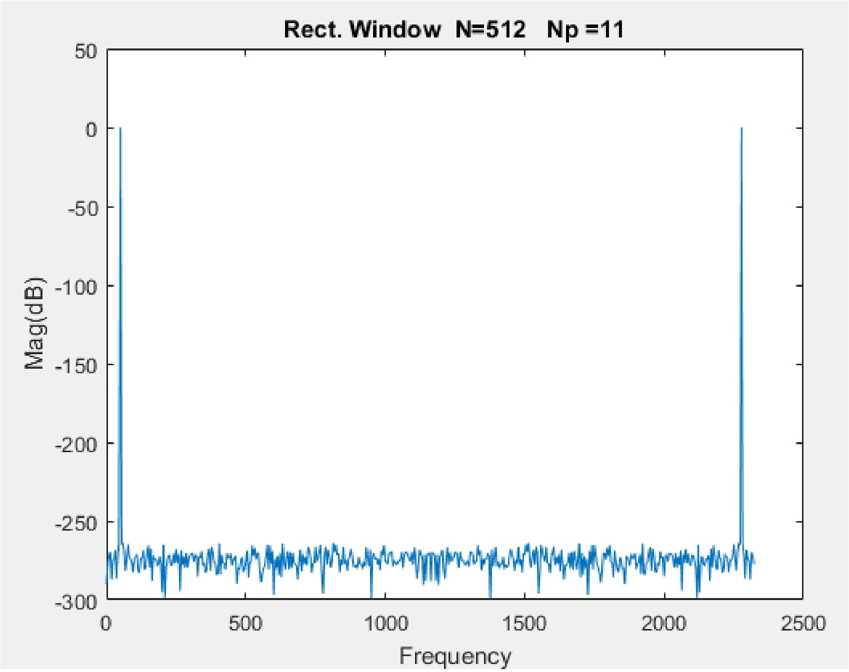
512 Samples with Strobe Period

```
Pyyt2 =

  Columns 1 through 12

 -289.9823 -277.1621 -271.8971 -269.7867 -287.1463 -274.9517 -274.1616 -268.2808 -286.3367 -270.6890 -264.2517    0.0000

  Columns 13 through 24

 -264.9125 -263.7218 -269.4627 -281.3076 -273.7005 -264.8333 -273.1021 -274.2804 -275.7987 -282.2465 -273.4191 -272.4758

  Columns 25 through 36

 -270.9356 -281.3558 -282.8219 -274.2676 -273.4832 -268.4204 -280.8477 -279.3164 -270.8593 -265.4445 -279.4813 -267.3563

  Columns 37 through 48

 -288.0473 -271.6637 -274.6875 -274.0074 -278.3424 -277.6395 -272.2091 -276.9768 -295.2068 -265.0774 -298.7341 -280.0345
```

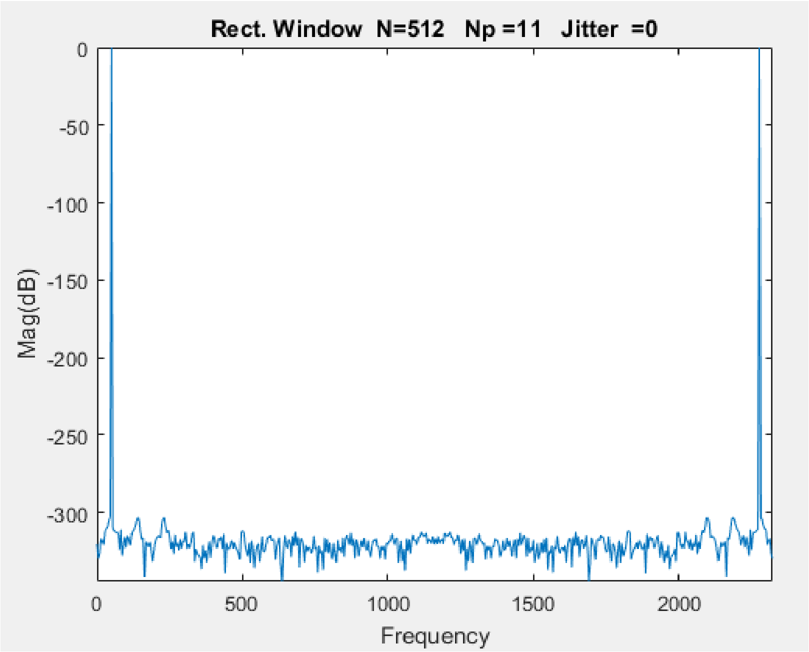
512 Samples with Strobe Period



MatLab comparison: 512 Samples with Strobe Period Sweep

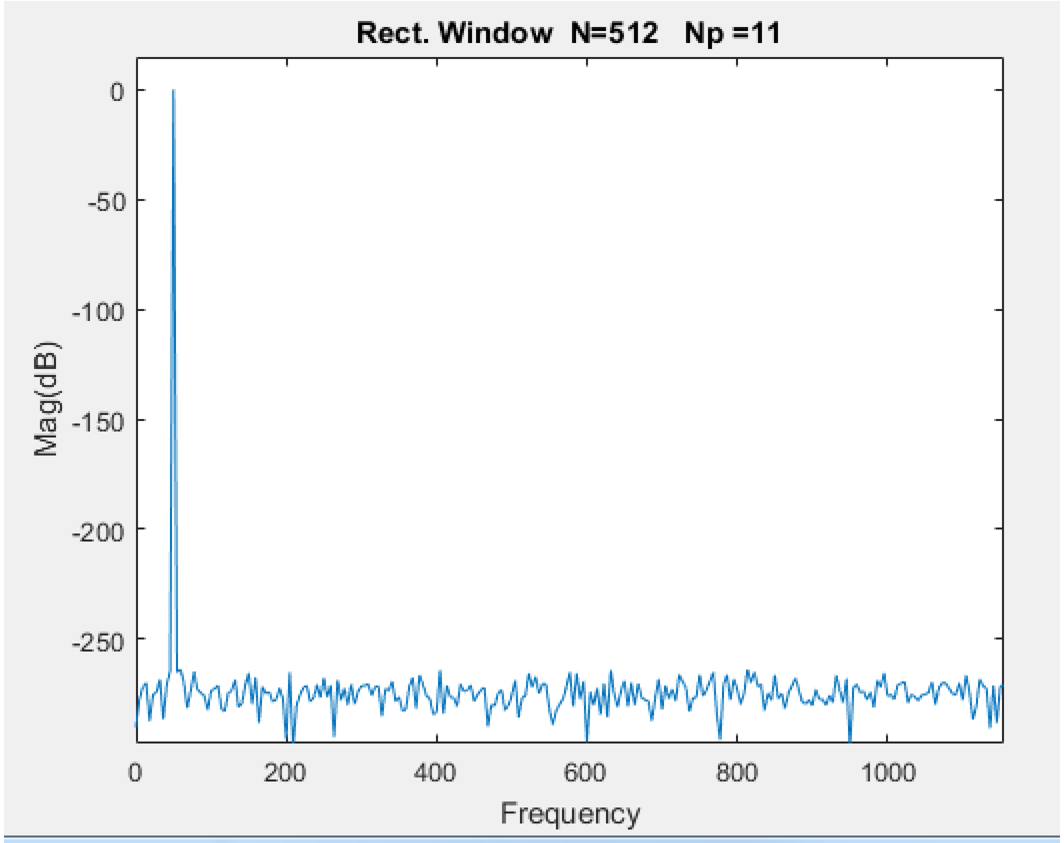


Spectre Results

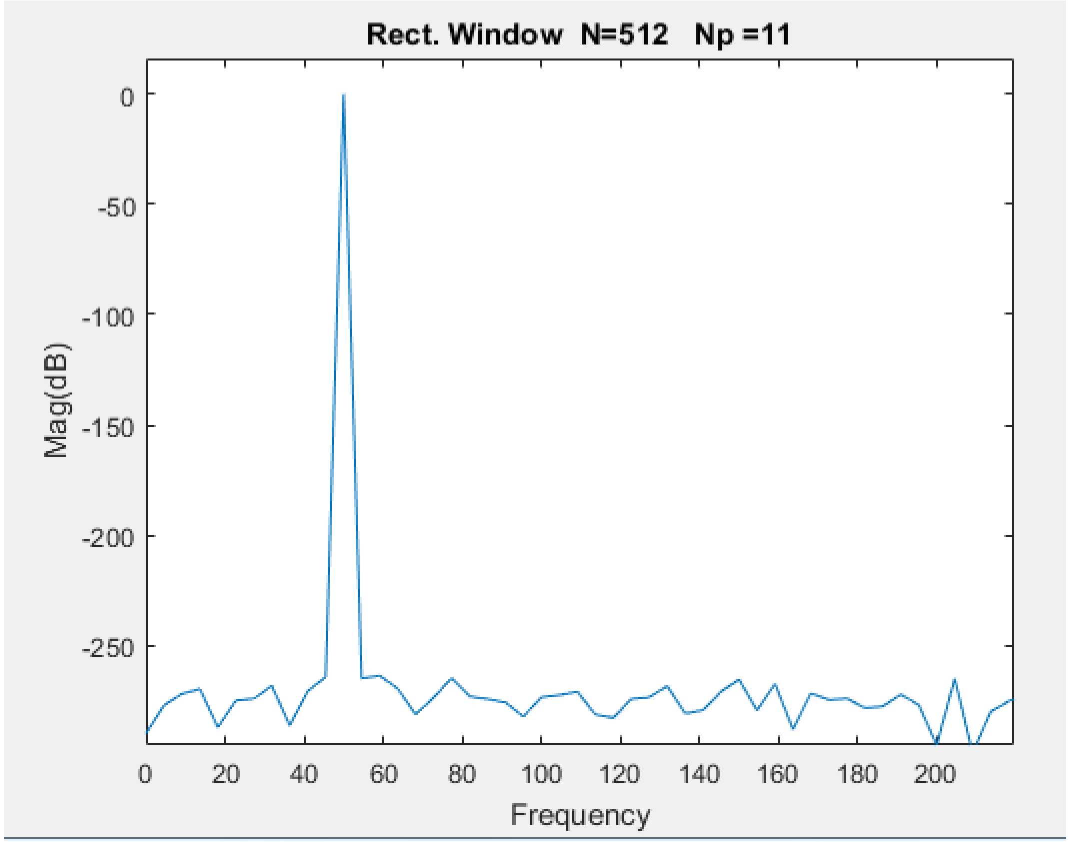


MatLab Results

512 Samples with Strobe Period



512 Samples with Strobe Period



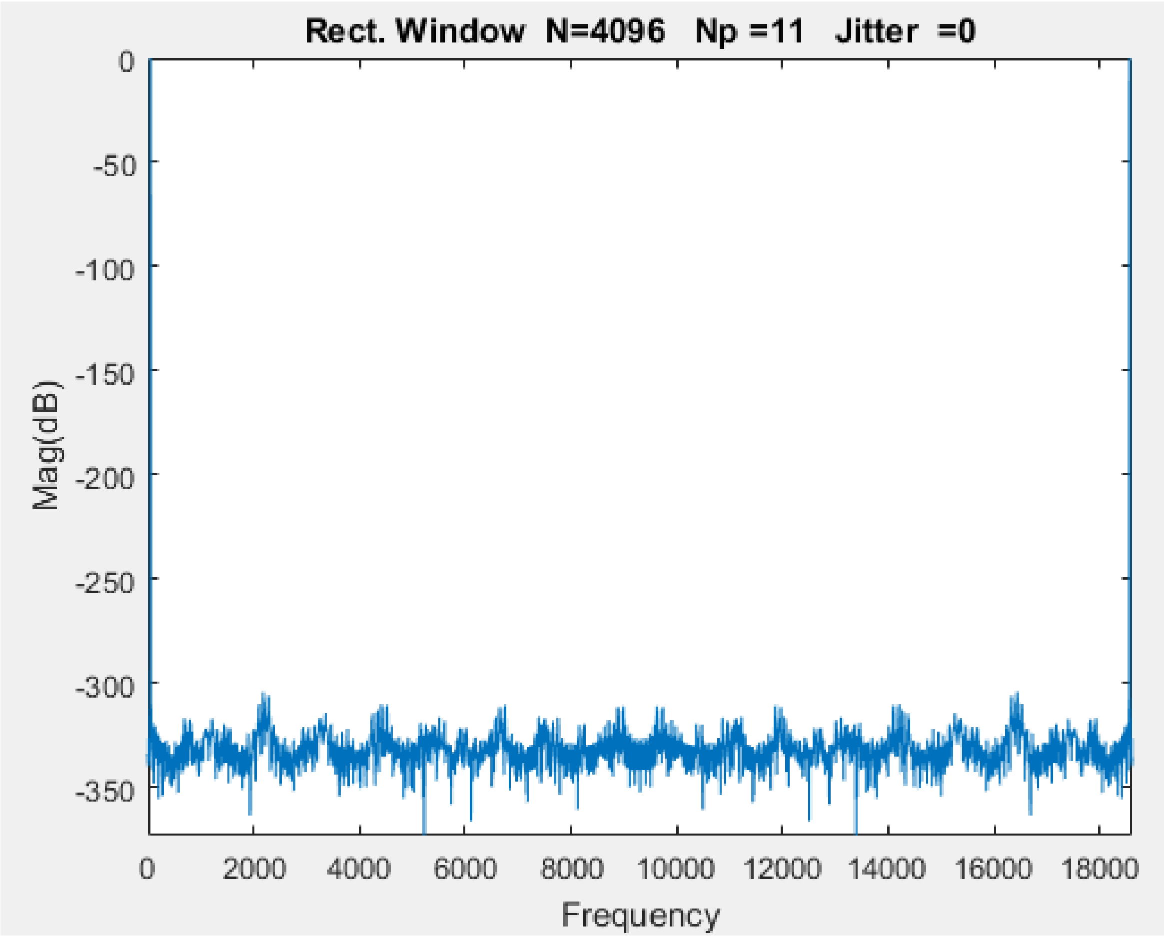
4096 Samples with Standard Sweep

$V(t)=\sin(2*\pi*50t)$

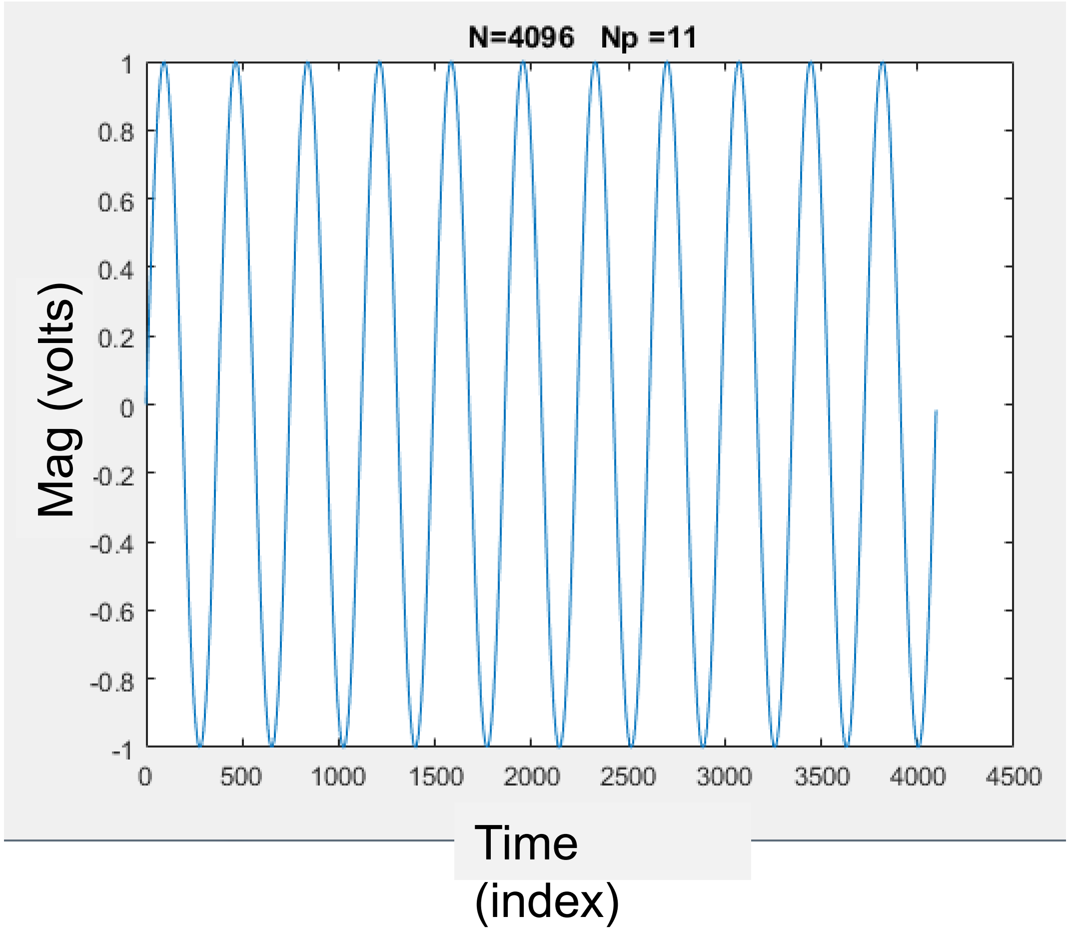
11 periods

Coherent Sampling

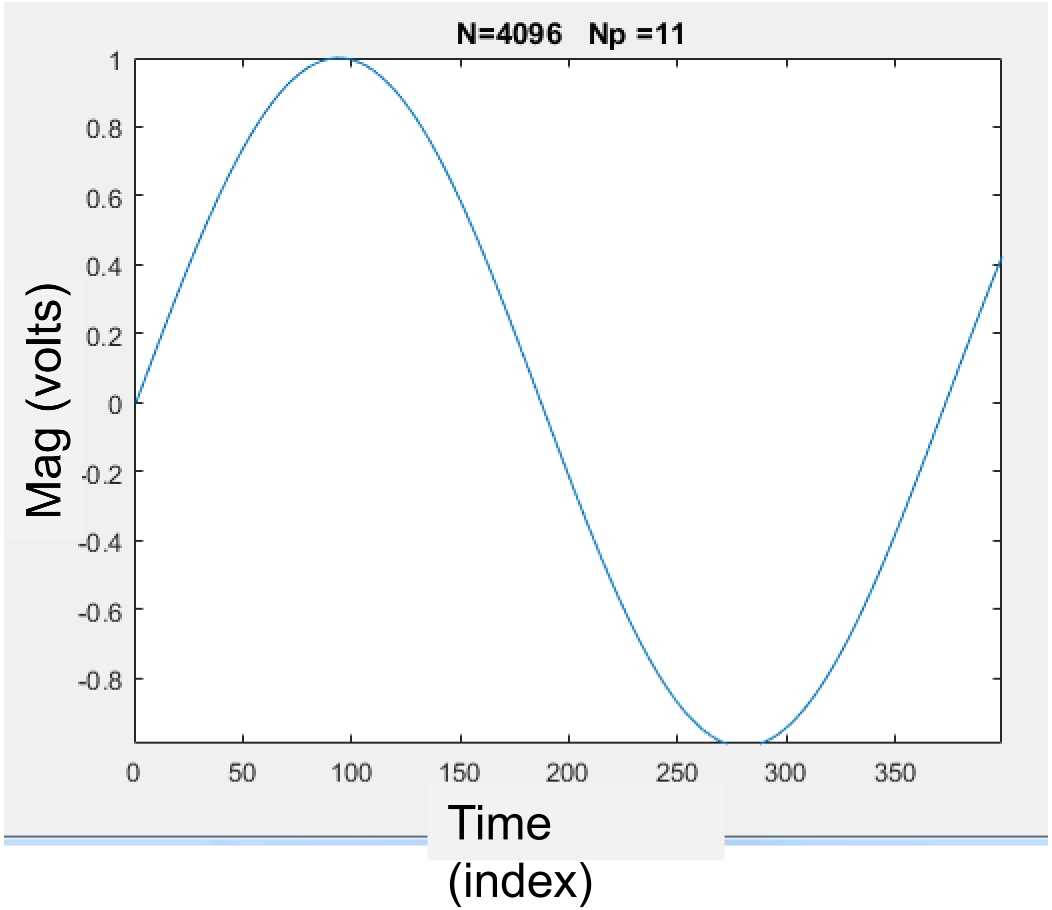
For reference: Results obtained with MatLab for N=4096



4096 Samples with Standard Sweep



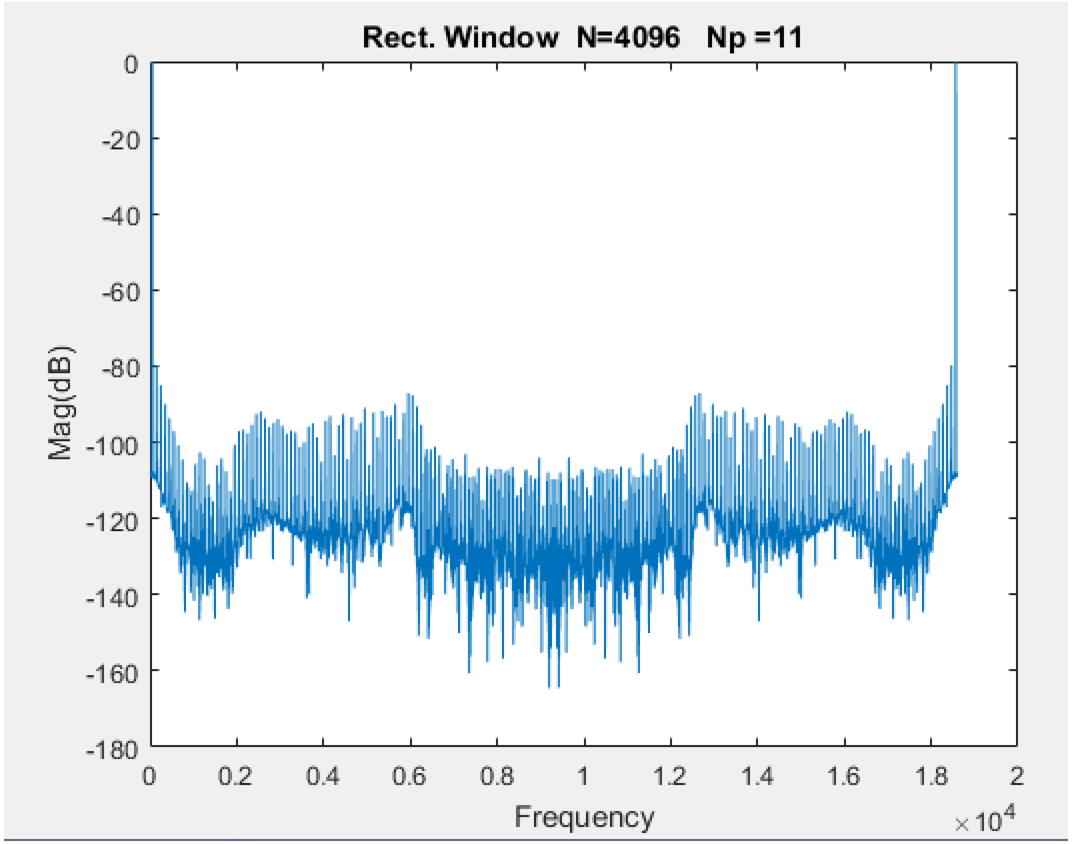
4096 Samples with Standard Sweep



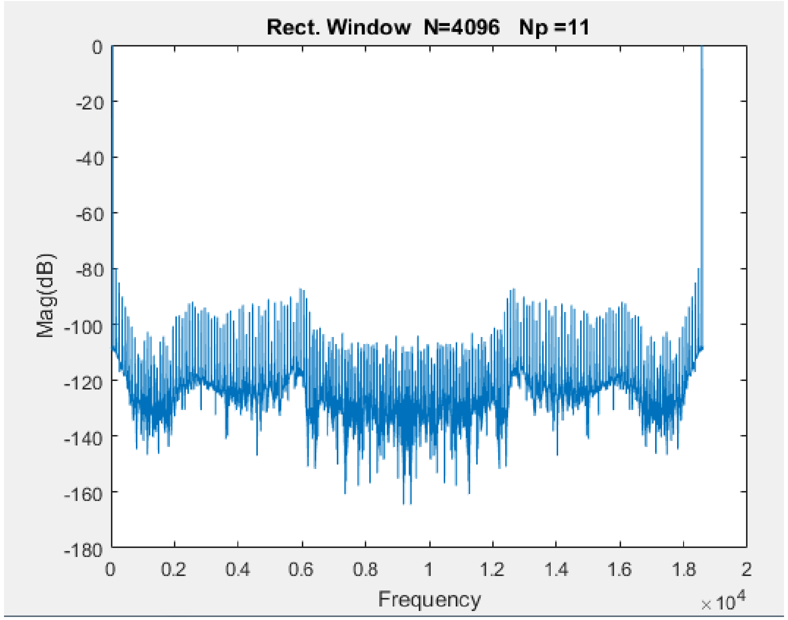
4096 Samples with Standard Sweep

```
Pyyt3 =

  Columns 1 through 12

 -108.3243 -108.1538 -108.2707 -108.0618 -108.1535 -108.0412 -108.0422 -109.2666 -107.9762 -107.8815 -107.9734   -0.0024

  Columns 13 through 24

 -108.0437 -108.0723 -108.1927 -109.4684 -108.4265 -108.3864 -108.7503 -107.7815 -109.1369 -109.1277 -109.4911 -109.7293

  Columns 25 through 36

 -109.7098 -108.8871 -109.7944 -109.4286 -109.8171 -108.5378 -109.8456 -109.8612 -109.9177  -79.7470 -110.0501 -110.1301

  Columns 37 through 48

 -110.2518 -104.6517 -110.5298 -110.8421 -110.8907 -111.4034 -111.3159 -111.0945 -111.7096 -111.7185 -111.9500 -111.0991
```

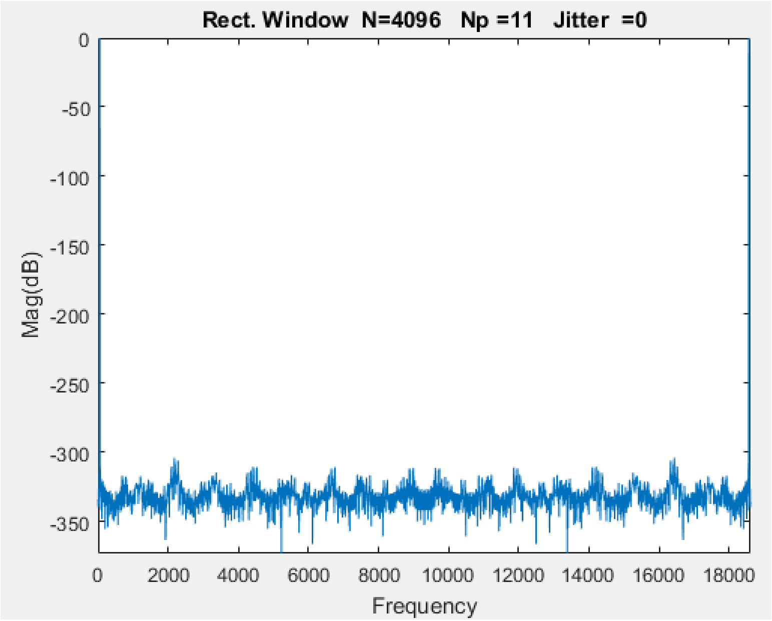
4096 Samples with Standard Sweep



Comparison 4096 Samples with Standard Sweep

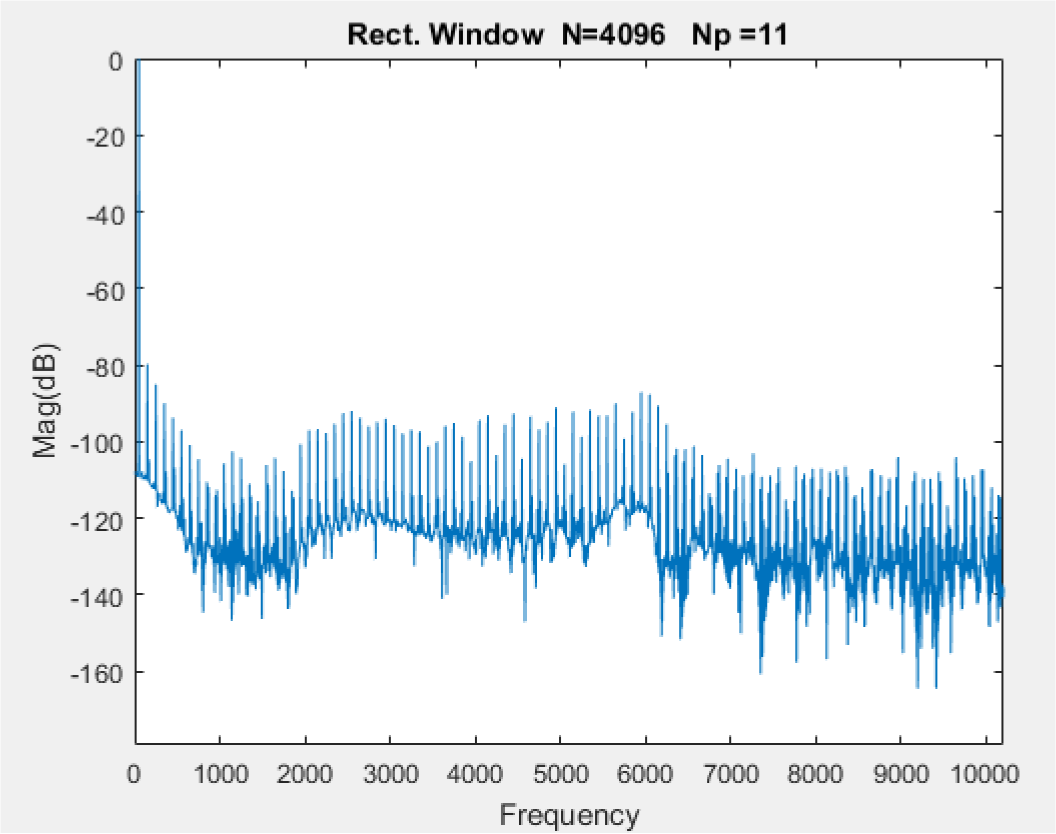


Spectre

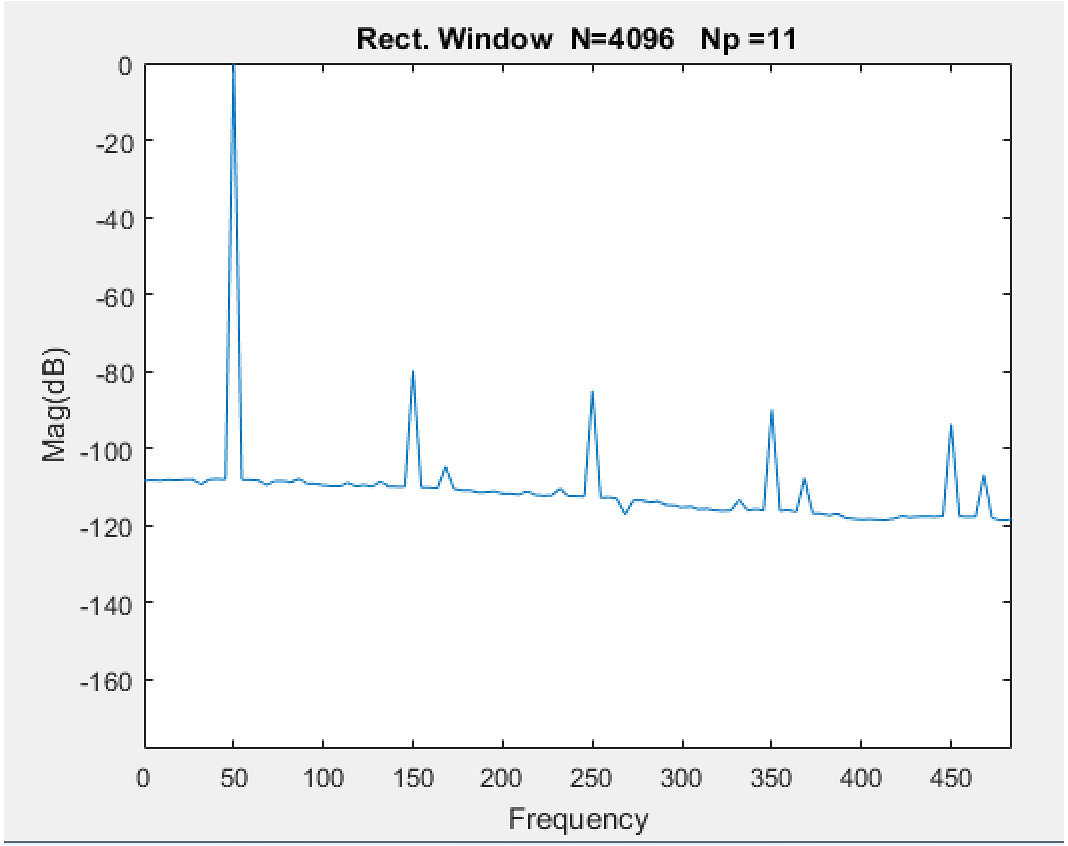


MatLab

4096 Samples with Standard Sweep

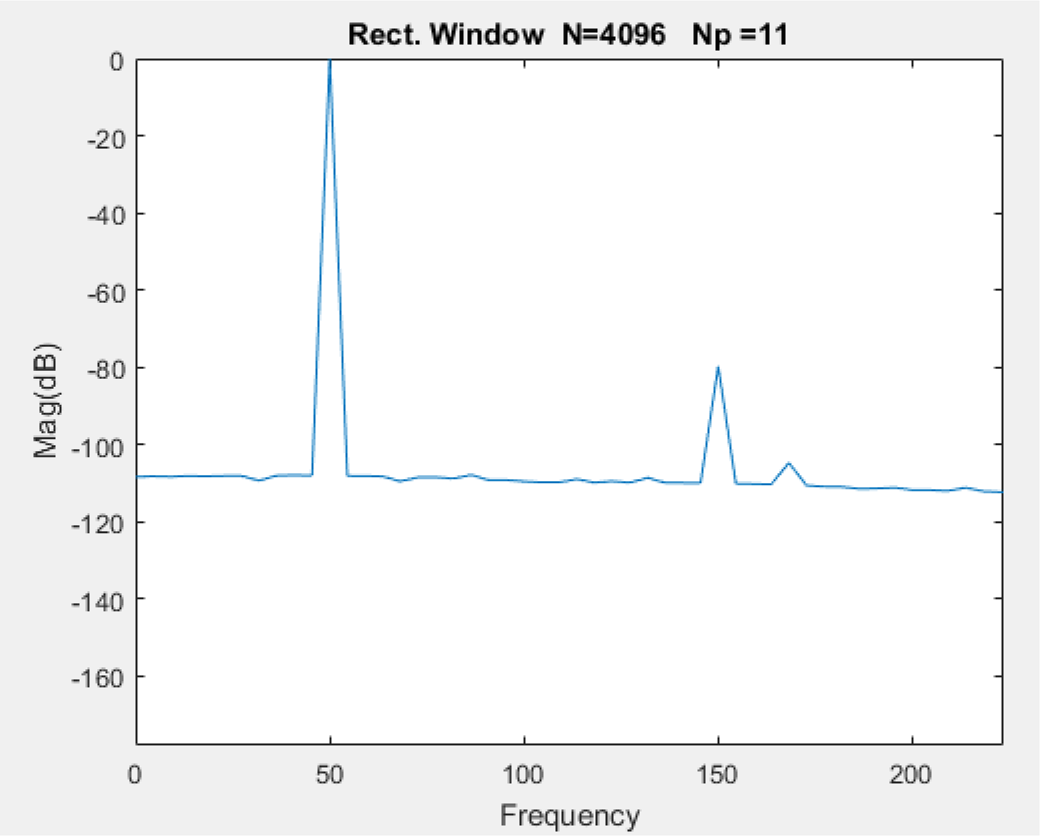


4096 Samples with Standard Sweep



Note presence of odd harmonics in spectrum

4096 Samples with Standard Sweep



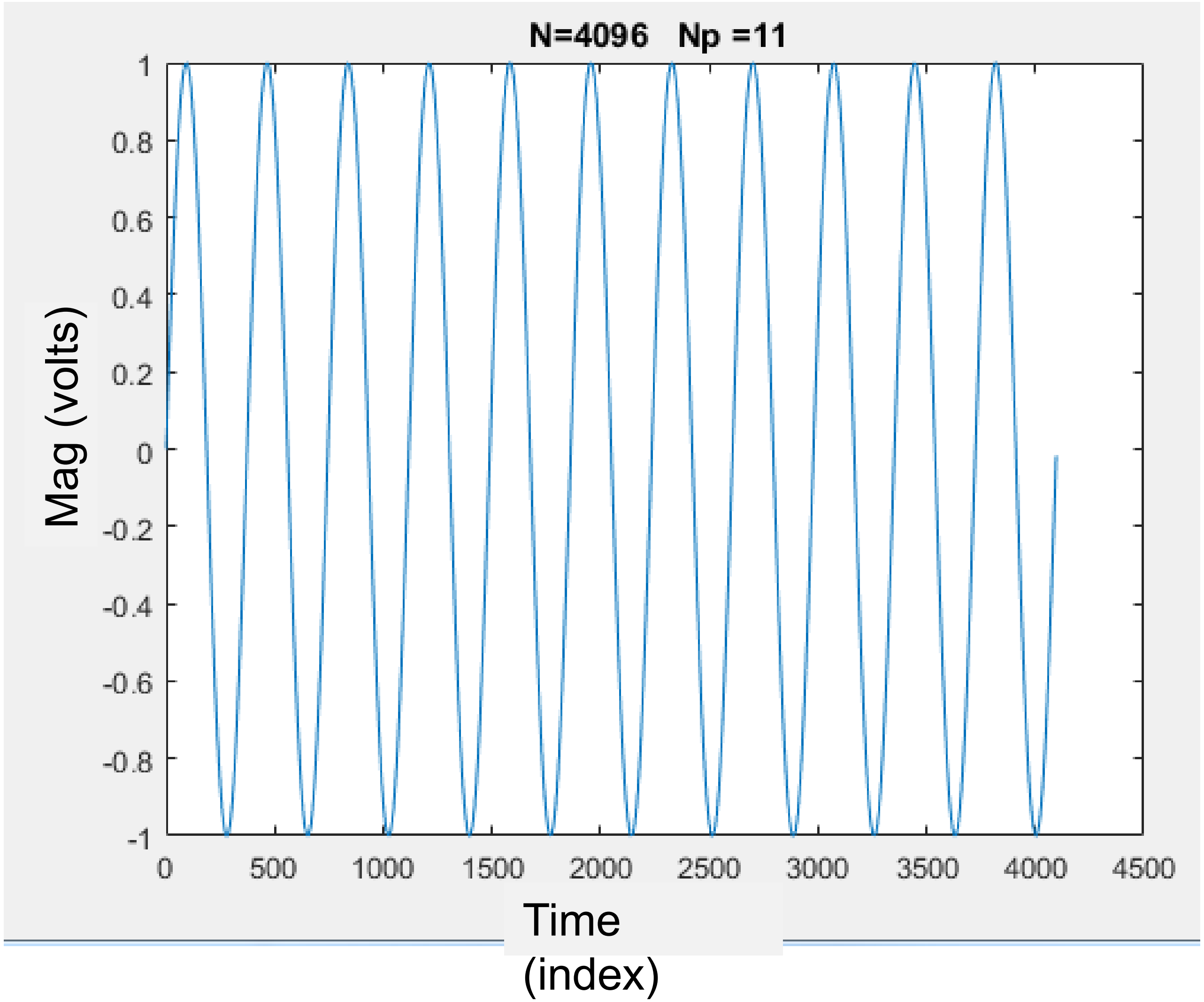
4096 Samples with Strobe Period Sweep

$V(t)=\sin(2*\pi*50t)$

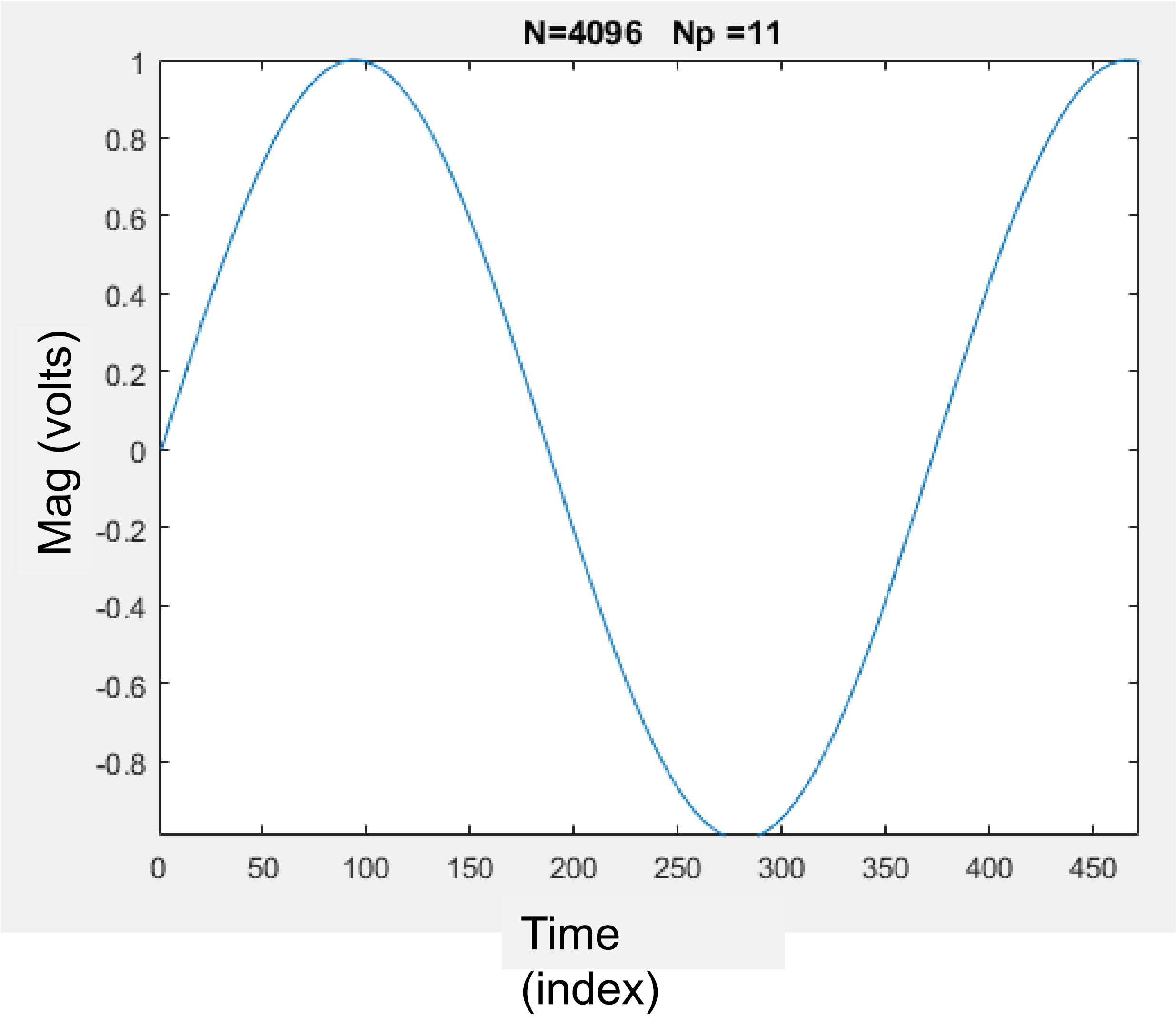
11 periods

Coherent Sampling

4096 Samples with Strobe Period



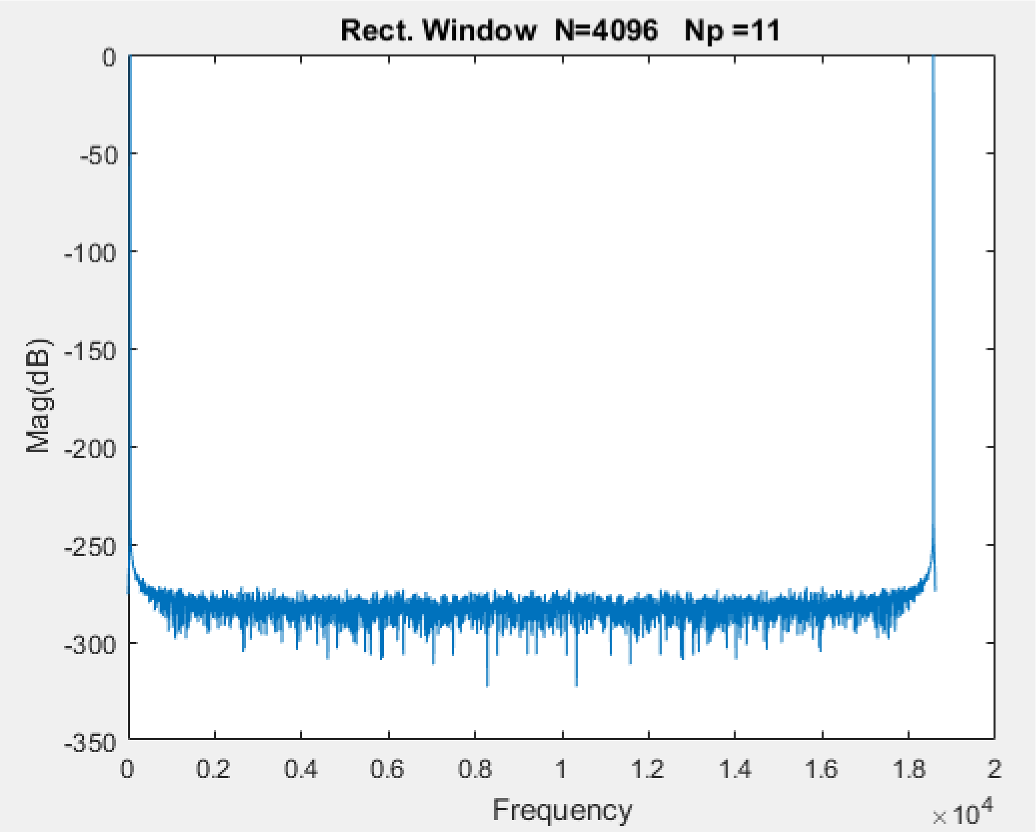
4096 Samples with Strobe Period



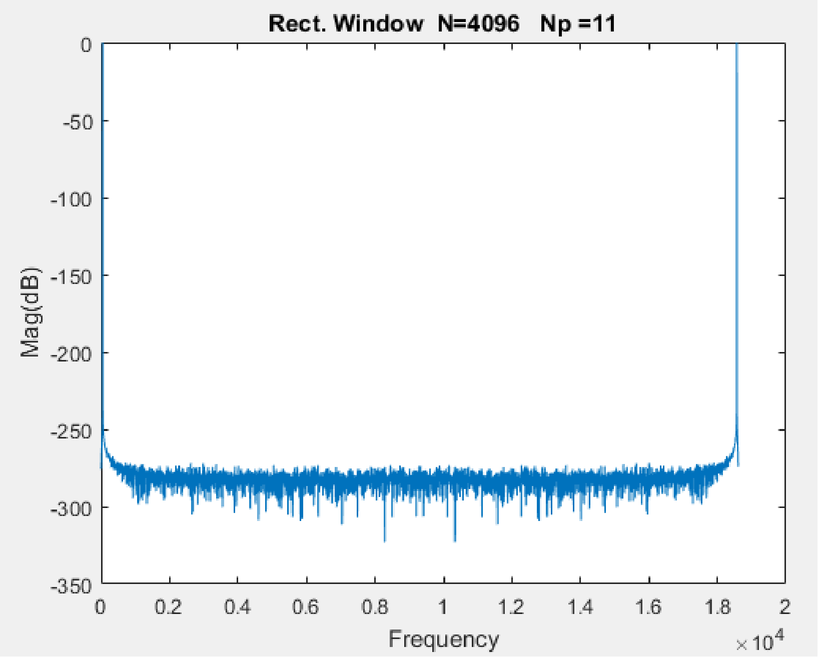
4096 Samples with Strobe Period

```
Pyyt4 =

  Columns 1 through 12

 -275.7993 -274.2288 -271.2122 -266.4879 -261.9356 -260.8993 -256.6778 -254.4684 -252.1418 -247.4997 -238.6334   -0.0000

  Columns 13 through 24

 -237.9355 -245.9934 -249.3333 -250.9084 -252.6585 -254.7659 -255.1826 -256.4114 -256.4550 -258.0311 -258.5182 -259.4258

  Columns 25 through 36

 -260.1189 -260.6618 -260.7491 -261.8162 -261.6129 -262.3357 -262.2738 -263.3625 -264.1235 -263.3313 -262.6442 -264.9796

  Columns 37 through 48

 -265.6690 -265.3833 -264.5215 -264.5995 -268.4617 -266.2074 -265.4619 -265.3134 -267.1584 -267.0717 -265.9300 -268.4336
```

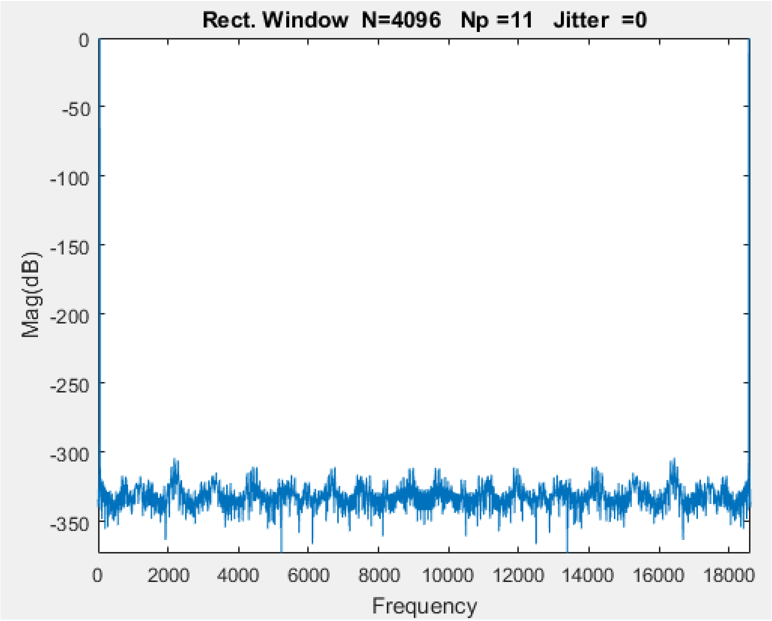
4096 Samples with Strobe Period



Comparison 4096 Samples with Strobe Period Sweep

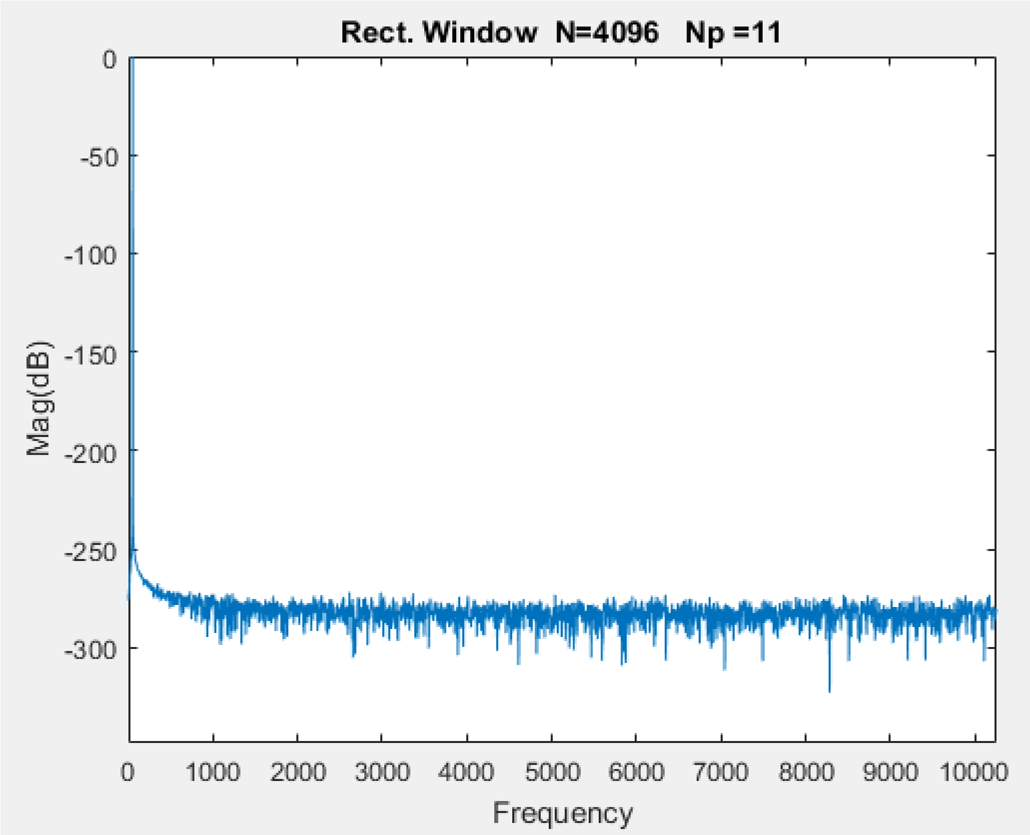


Spectre

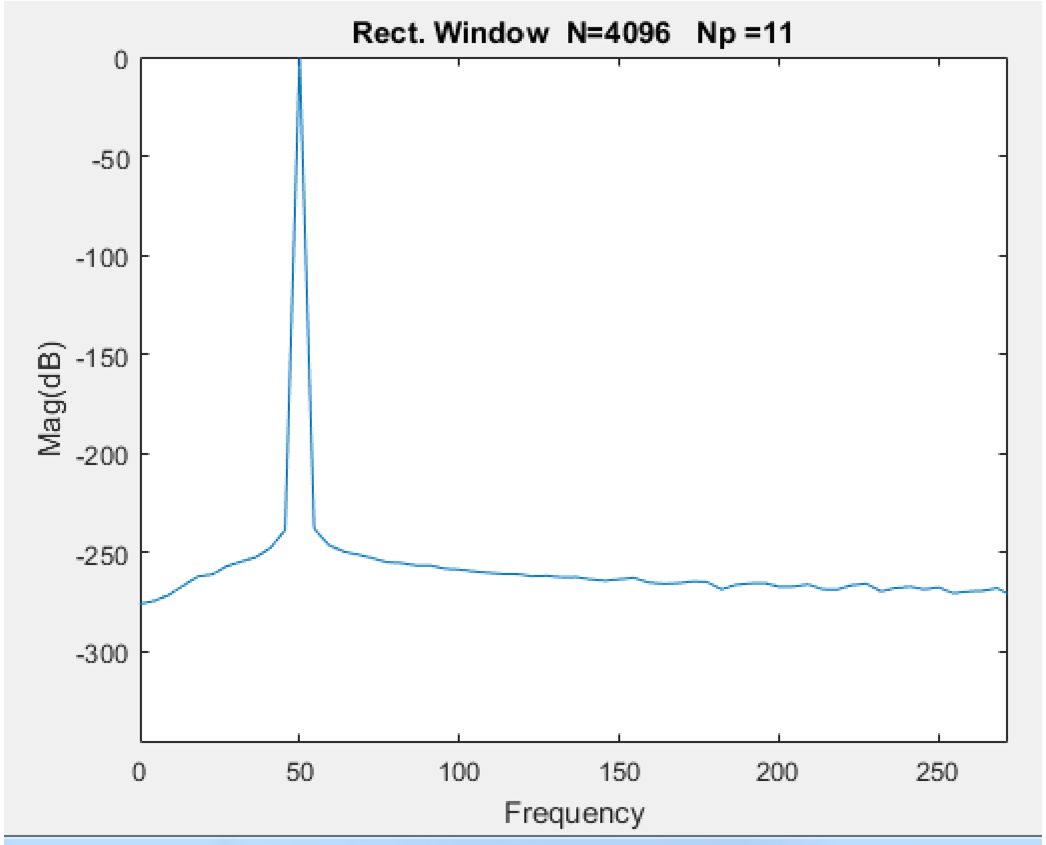


MatLab

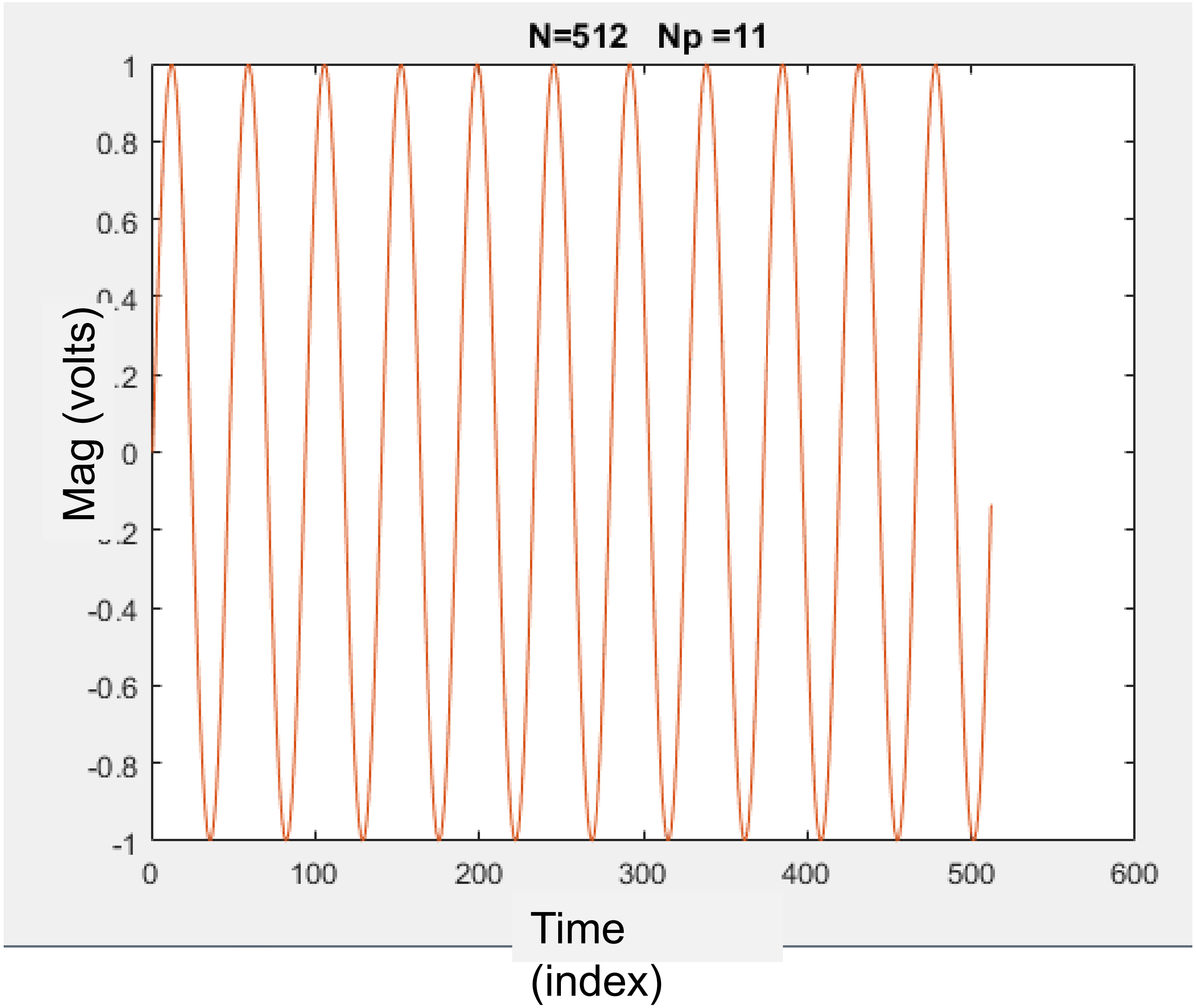
4096 Samples with Strobe Period



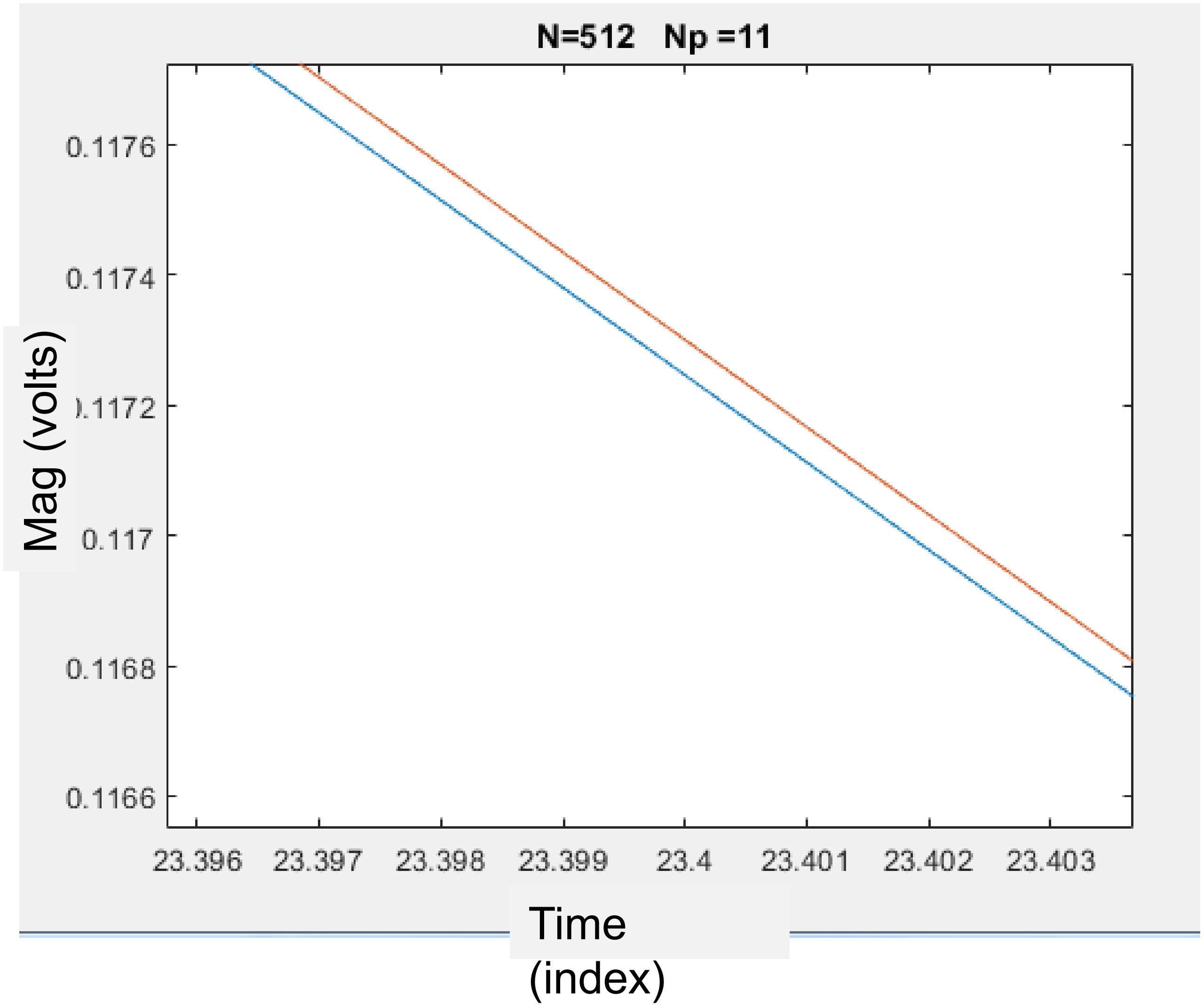
4096 Samples with Strobe Period



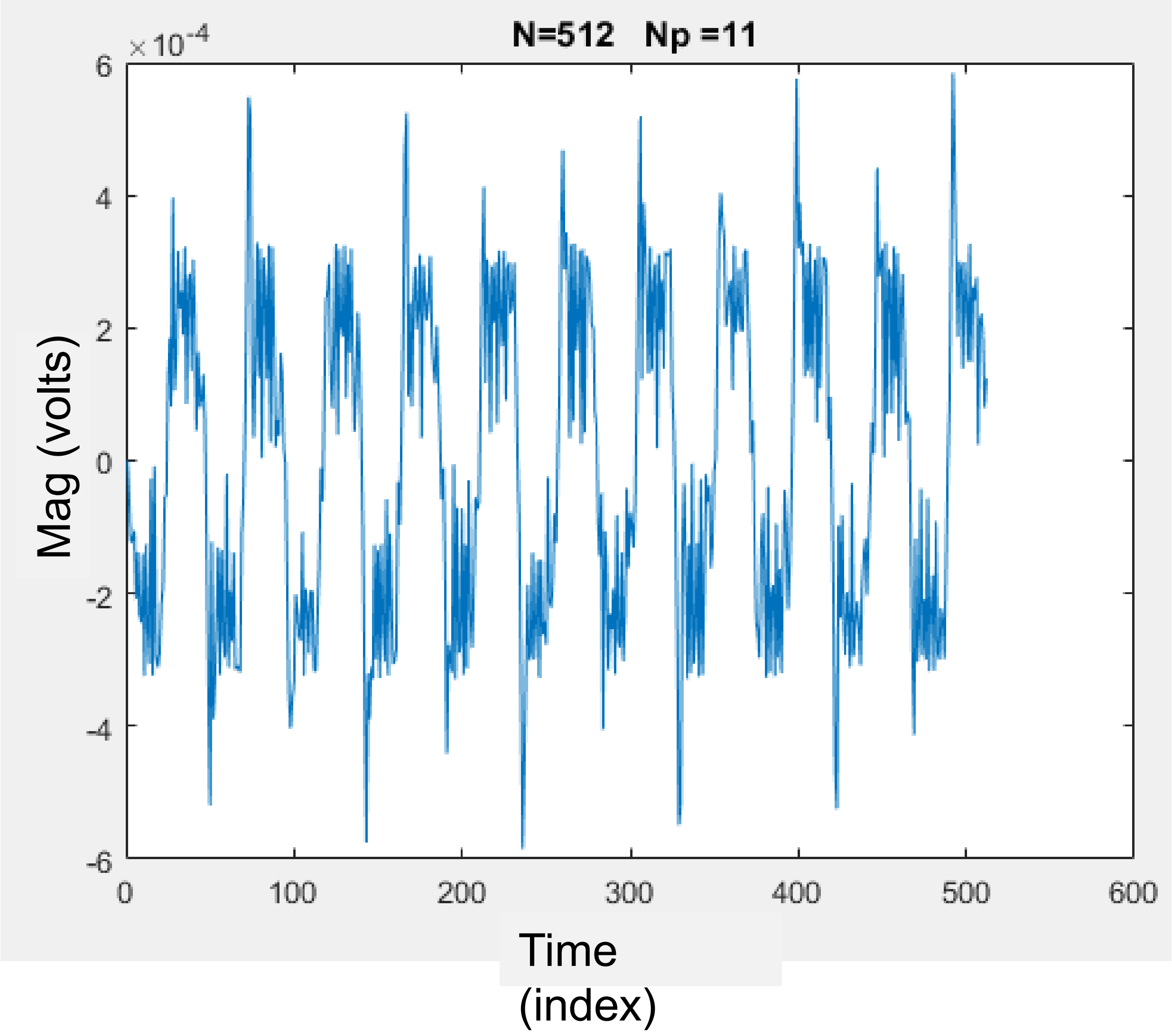
Superimposed Standard/Strobe Sweep



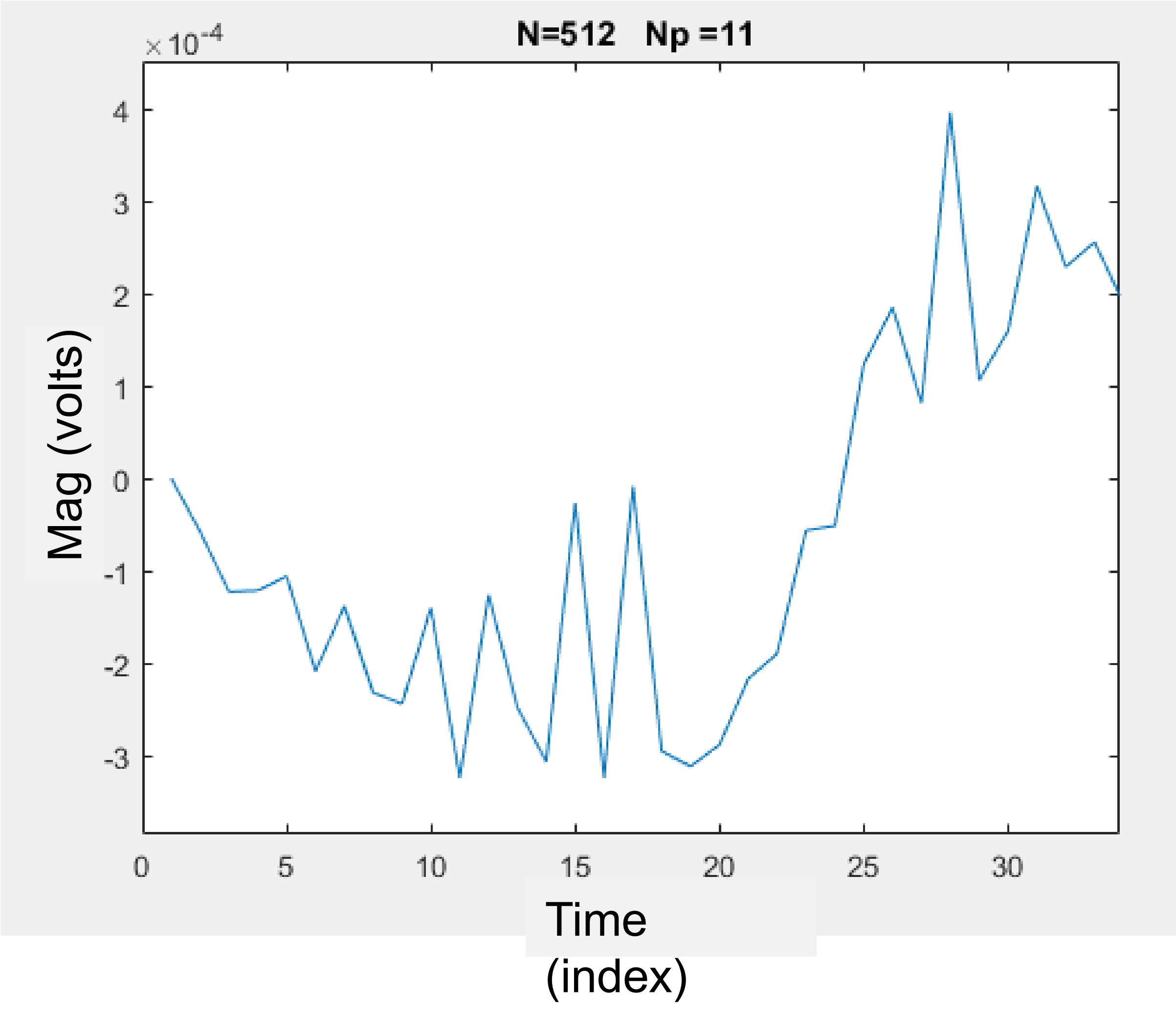
Superimposed Standard/Strobe Sweep



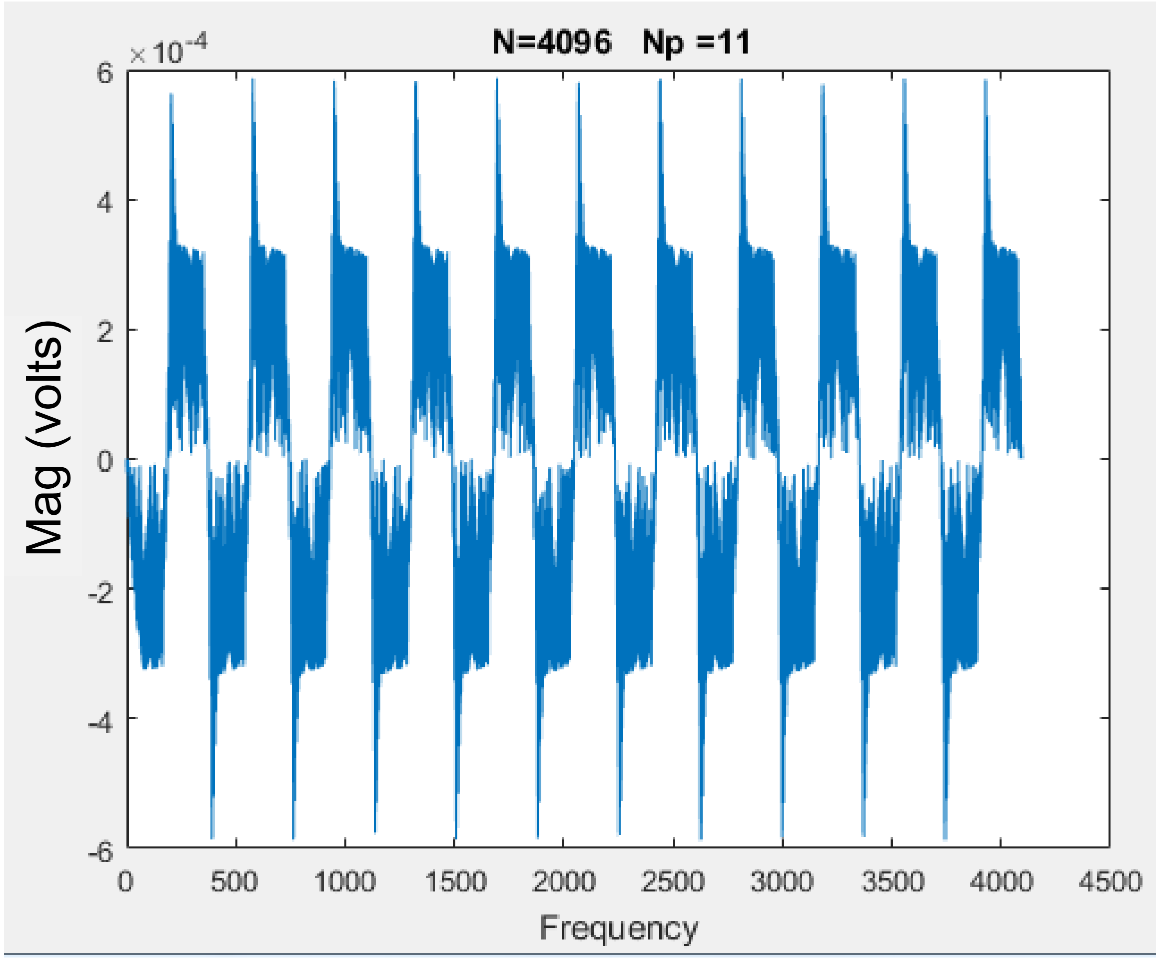
Difference Standard/Strobe Sweep



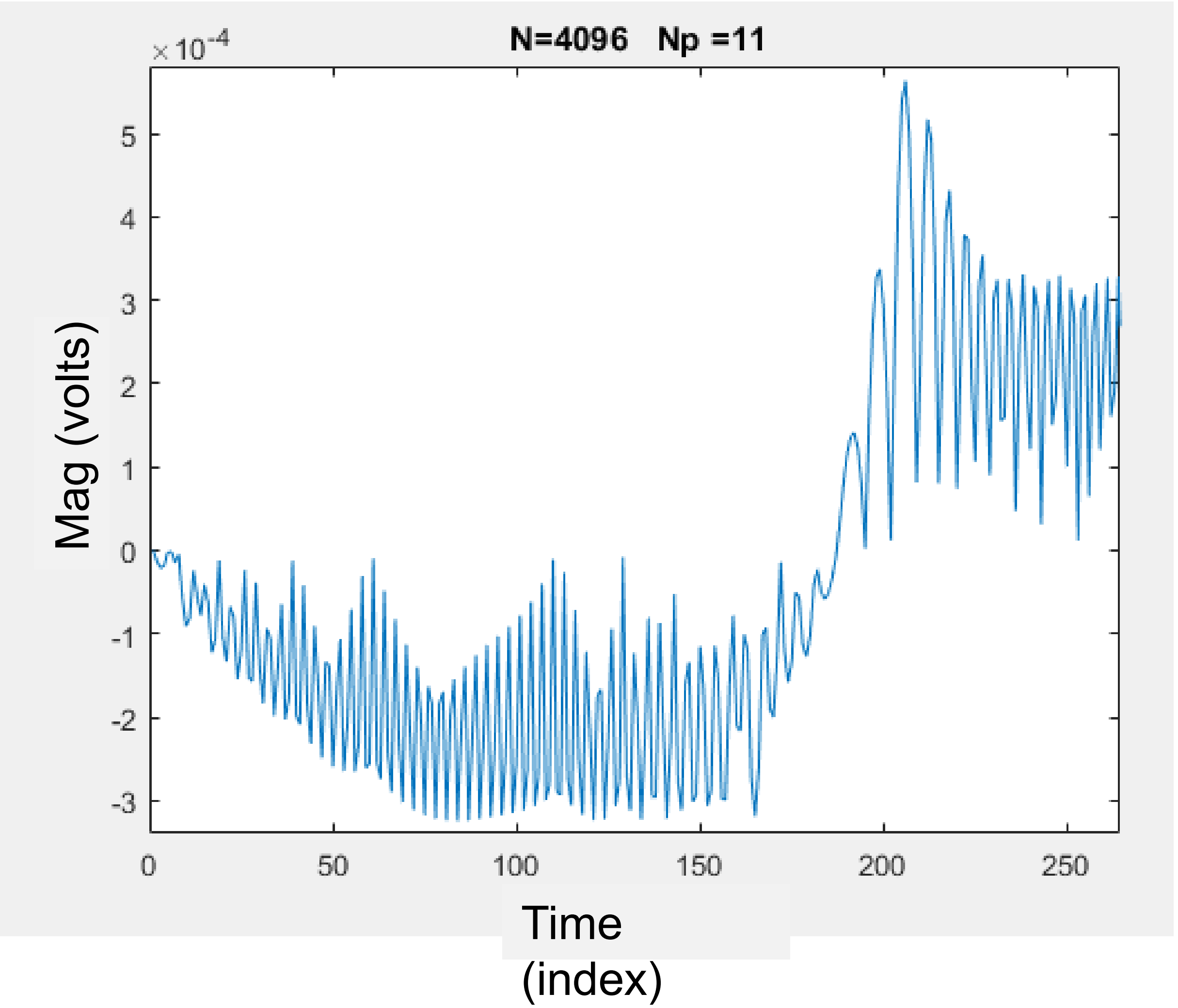
Difference Standard/Strobe Sweep



Difference Standard/Strobe Sweep



Difference Standard/Strobe Sweep



Addressing Spectral Analysis Challenges

- Problem Awareness
- Windowing and Filtering
- Post-processing

Problem Awareness

THEOREM: Consider a periodic signal with period T=1/f and sampling period $T_S=1/f_S$. If N_P is an integer, x(t) is band limited to f_{MAX}, and $f_s>2f_{max}$, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad 0 \le m \le h-1$$

and $X(k)=0$ for all k not defined above

where $\left\langle X(k)\right\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\left\langle x(kT_S)\right\rangle_{k=0}^{N-1}$

N=number of samples, N_P is the number of periods, and $h = \mathrm{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

- Hypothesis is critical
- Even minor violation of the premise can have dramatic effects
- Validation of all tools is essential
- Learn what to expect

Filtering - a strategy to address the aliasing problem

- A lowpass filter is often used to enforce the band-limited requirement if not naturally band limited
- Lowpass filter often passive
- Lowpass filter design often not too difficult
- Minimum sampling frequency often termed the Nyquist rate.

End of Lecture 6